

# **A Miscellany of Extracts from a Dictionary of Mathematics**

There are 14 double-page spreads (= 7 sheets)  
covering the following topics:

**eponyms**

**famous problems**

**polygons**

**quadrilaterals**

**recreational mathematics (1)**

**recreational mathematics (2)**

**symbols and their origins**

The sample pages printed here are reproduced from the  
*Oxford Mathematics Study Dictionary*  
by *Frank Tapson*

Published (in the UK) by the  
Oxford University Press  
ISBN 0-19-914567-9  
(Second edition published in July 1999)

The Dictionary is printed in black and red as shown here on screen.

It is arranged on a thematic basis, and more details of this and a full list of the themes are given here on the last page.

A slight reduction in size has been made in this presentation from that of the original book in order to fit 2 pages on a single side of A4 paper. Also a few entries have been modified in order to fit the reduced space available.

The permission of the publishers to present this sample is gratefully acknowledged.

**eponyms**

**eponym** An eponym is EITHER the name of a person, factual or fictitious, which is used to form a word or phrase identifying a particular thing OR it is the thing itself. *The person's name is usually that of the one first associated in some way with whatever is being identified. The name may, or may not, be that of the originator. Other eponyms are given under separate topics.*

**Eratosthenes' sieve** is an **algorithm** for finding **prime numbers**. First write down as many numbers as required to be searched, in order, starting with 1 and not missing any out. Cross out 1. Leave 2 and cross out every 2nd number (4, 6, 8 etc). Leave 3 and cross out every 3rd number (6, 9, 12 etc). 4 is already crossed out, so leave 5 and cross out every 5th number (10, 15 etc). When complete the numbers left are the prime numbers.

<del>1</del>	2	3	<del>4</del>	5	<del>6</del>
7	<del>8</del>	<del>9</del>	<del>10</del>	11	<del>12</del>
13	<del>14</del>	<del>15</del>	<del>16</del>	17	<del>18</del>
19	<del>20</del>	<del>21</del>	<del>22</del>	23	<del>24</del>
<del>25</del>	<del>26</del>	<del>27</del>	<del>28</del>	29	<del>30</del>
31	<del>32</del>	<del>33</del>	<del>34</del>	<del>35</del>	<del>36</del>
37	<del>38</del>	<del>39</del>	<del>40</del>	41	<del>42</del>
43	<del>44</del>	<del>45</del>	<del>46</del>	47	<del>48</del>
<del>49</del>	<del>50</del>	51			

**Euclid's algorithm** is a method to find if two numbers have a common factor, and its value, if there is one.

Consider two numbers  $M$  and  $N$  with  $M > N$   
 Let  $P = M$  and  $Q = N$   
 → Divide  $P$  by  $Q$  and find the remainder  $R$   
 If  $R = 0$  then  $Q$  is a factor of  $M$  and  $N$   
 If  $R = 1$  then  $M$  and  $N$  have only 1 as a common factor  
 If  $R > 1$  then put  $P = Q$  and  $Q = R$  and go to →

**Fermat's last theorem** is that the equation  $x^n + y^n = z^n$  has no solutions in whole numbers for  $x, y$  and  $z$  if  $n > 2$  and  $x, y, z > 1$ , and it is one of the most famous theorems in mathematics. It was really a conjecture since, although it was known about in the 1600's, it was not proved until 1994.

**Fermat's problem** is to find the point F in any triangle ABC, that makes the total of the distances AF + BF + CF to be the least. It is also known as **Steiner's problem**.

**Gaussian integer** A Gaussian integer is a **complex number** in which both the real and imaginary parts are whole numbers.  
*Examples:  $3 + 8i$  and  $10 + 2i$  are Gaussian integers;  $7.6 + 5i$  is not.*

**Goldbach's conjecture** is that every even number from 6 onwards can be made by adding two **prime numbers**. There are often several possibilities.  
*Examples:  $6 = 3 + 3$ ;  $10 = 3 + 7$  &  $5 + 5$ ;  $22 = 3 + 19$  &  $5 + 17$  &  $11 + 11$*

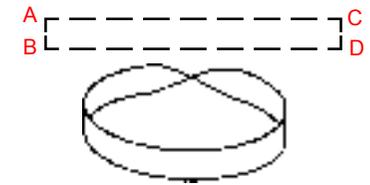
**Hamiltonian walk** A Hamiltonian walk is a path traced out on a **topological graph** which visits every vertex once and once only - except possibly for the start and finish which might be on the same vertex.

**Heronian triangle** A Heronian triangle is a triangle whose three edge lengths and its area are all **rational numbers**.

**Mersenne primes** are those **prime numbers** which can be made from the expression  $2^n - 1$ . It can only be true when  $n$  itself is prime but even then there are times when it is not true. For instance, it works when  $n = 2, 3, 5$  or  $7$  but not when  $n = 11$  or  $23$ , as well as many other prime values.

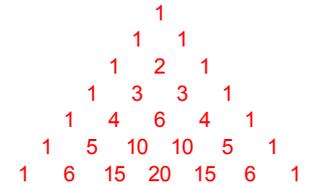
**eponyms**

**Mobius band or strip** A Mobius band is made by taking a rectangular strip of paper like that shown as ABCD; fastening the two shorter edges together (AB and CD) BUT BEFORE FASTENING, giving the strip a half twist so that A fastens to D and B to C. The strange property of this band is that it now has only 1 edge and 1 side.

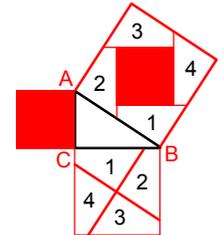


**Pascal's triangle** is an array of numbers in the shape of a triangle, having a 1 at the top and also at the ends of each line. All the other numbers are made by adding the pair of numbers closest to them in the line above.

*Examples:  $1 + 4 = 5$     $4 + 6 = 10$     $6 + 4 = 10$     $4 + 1 = 5$*

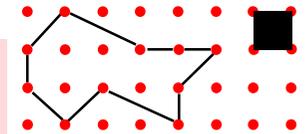


**Perigal's dissection** is a visual way of illustrating **Pythagoras' theorem**. In the diagram on the right, cutting out and moving the necessary pieces shows how the square drawn on the hypotenuse AB is made up of the (red) square drawn on the shorter edge AC plus the square drawn on the other edge BC which is divided into the four numbered quadrilaterals.



**Pick's theorem** A grid of dots is marked out in a square array, then a polygon is drawn on this grid by joining up dots with straight lines to make the edges of the polygon, with none of these edges crossing.

Let the number of dots inside the polygon =  $I$   
 Let the number of dots on the boundary =  $B$   
 Then the area of the polygon is  $(\frac{1}{2} B + I - 1) \times \text{area of a unit square}$ .



*Example: In the polygon drawn above,  $B = 10$   $I = 4$ , so its area is  $(5 + 4 - 1) = 8$  times the area of the unit square which is coloured black.*

**Pythagorean triplets** are groups of three numbers ( $a, b, c$ ) which satisfy the equation  $a^2 + b^2 = c^2$  Basic triplets can be easily made this way:

Choose two numbers  $m, n$  which have NO common factors (except 1)  
 Then  $a = m^2 - n^2$     $b = 2mn$     $c = m^2 + n^2$

*From any basic triplet others can be made by multiplying each of its three numbers by some constant. So (3, 4, 5) gives (6, 8, 10) or (21, 28, 35) etc.*

**Zeno's paradoxes** are concerned with motion and some apparent impossibilities. They were important in the development of mathematical thinking. There are four paradoxes, but the best known is that of Achilles and the tortoise. Achilles races against a tortoise and, since Achilles is clearly the faster of the two, gives the tortoise a good start. Now as soon as Achilles gets to the point at which the tortoise started, the tortoise has moved, and this is repeated over and over. How can Achilles ever catch up with the tortoise?

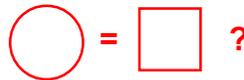
## famous problems

Throughout the history of mathematics there have been particular problems which have not only been important in themselves but also, because of the interest they attracted. Important, that is, for the amount of new mathematics generated as people tried either to solve them or, just as importantly, prove that they could not be solved. **Fermat's last theorem** is perhaps the most famous of all.

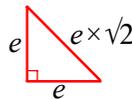
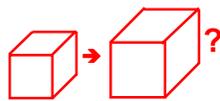
**geometrical constructions** are accurate diagrams drawn as an answer to a problem, using only a **pair of compasses** and a **straight-edge**. The ancient Greeks, with whom much of our mathematics started, were mainly interested in geometry. To them, most problems were expressed in terms of geometry and their solutions were restricted to being geometrical constructions. So, to divide a given straight line into equal parts, it was not acceptable to do it by measuring, it had to be done by a geometrical construction.

**trisection of an angle** One of the earliest **geometrical constructions** devised was that for the bisection of an angle - dividing it into two equal parts. Not unreasonably, mathematicians then sought to find a way of trisecting an angle - dividing it into three equal parts. Not until the 19th century was that proved to be impossible.

**squaring the circle** A very early problem required a square to be made, using a **geometric construction**, that was equal in area to a given circle. This meant finding a length  $e$  for the edge of the square so that  $e^2 = \pi r^2$  where  $r$  is the radius of the circle. This means  $e = r\sqrt{\pi}$ . For over 2000 years mathematicians struggled with this problem. Many close approximations were found, but never an exact solution. In 1882 it was finally proved that it was impossible by any geometrical construction.



**duplication of a cube** In ancient times, just as now, problems were dressed up in the form of a story. One concerned the temple at Delphi (in Greece) where there was a famous oracle which was often consulted for advice. The story said that, in order to avert a plague, the oracle had required that a new stone altar should be made in the shape of the original one (a cube) but having twice the volume. In modern terms, starting with a cube whose length of edge is  $e$ , the cube to be made has to have an edge of length  $k$  so that  $k^3 = 2e^3$  or  $k = e\sqrt[3]{2}$ . As with the **trisection** problem, at first sight it doesn't look difficult. In two dimensions the equivalent problem is to draw a square which is twice the area of a given square. In that case  $k = e\sqrt{2}$  and  $k$  is easily constructed as shown. Not until the 19th century was it proved that there was no possible **geometrical construction** for  $k = e\sqrt[3]{2}$ .

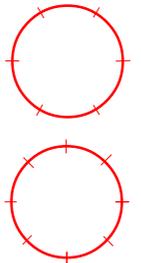


It is also known as the **Delian problem**.

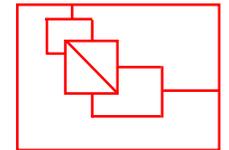
**Mascheroni constructions** The Italian mathematician Lorenzo Mascheroni (1750-1800) proved that any **geometrical construction** which was possible with compasses and straight-edge could also be done using only a pair of compasses. However, knowing that it can be done is one thing, finding how to do it is yet another problem.

## famous problems

**cyclotomy** is the topic concerned with dividing the circumference of a circle into equal parts, using only **geometrical constructions**. The early Greek mathematicians knew it was possible for all cases where the number of divisions was  $2^n$ , 3 or 5 and all other numbers obtained by multiplying any two of those together. So it was possible for 2, 3, 4, 5, 6, 8, 10, 12, 15 ... divisions. The problem of what other divisions were possible was unresolved until **Gauss** (who started on the problem when he was 19) proved that it was possible to construct  $2^{2^n} + 1$  divisions provided only that the expression yielded a prime. That added 17; 257 and 65537 ( $n = 2, 3, 4$ ) to the list.



**four colour problem** In the mid-1800's mathematicians became aware of a map-colouring problem which can be stated simply as "For any map that might be drawn on a flat surface, what is the least number of colours that are needed to colour it in such a way that no two countries which touch along a common border have the same colour?" Everyone who attempted it became convinced the answer was 4, but no one could prove it. It was not until 1976 (over 100 years after the problem was first stated) that a proof was found. *That meant it could then be called the four colour theorem. The drawing on the right shows just how complicated the 'maps' can become, but always 4 colours are enough - provided they are applied correctly.*



**bridges of Königsberg** In the 1600's there was a city called Königsberg (it is now called Kaliningrad). The city's buildings were spread over the banks of a river and an island, with all the four principal areas of land connected by 7 bridges. It was said to be a standard challenge to devise a route, starting anywhere, that would cross each bridge once and once only. It was generally believed to be impossible, but no one could prove it could not be done. In 1736 someone gave the problem to **Euler** who proved no answer was possible and, in doing so, started the branch of mathematics now known as **topology**. *It is of interest to consider how the problem would have been changed if there had been six or eight bridges.*



**factorising a number** is the process by which a whole number is broken up into two (or more) numbers which can be multiplied together to make the original number. Usually it is the **prime factors** which are sought. So, 2001 can be factored into  $3 \times 23 \times 29$ . Finding ways of doing this has fascinated mathematicians for almost as long as there has been arithmetic. Nowadays the problem concerns the finding of faster methods that can be used, with a computer, to find the factors of very big numbers (typically over 100 digits long) in as short a time as possible - to do it in hours or days, rather than in months or years. *This problem has become of practical interest in recent years because of modern encryption methods which use very large numbers (possibly as much as 200 digits long) which have only two prime factors.*

## polygons

**polygon** a polygon is a plane (=flat) shape completely enclosed by three or more straight edges. Usually edges are not allowed to cross one another, and the word is not often used for shapes having less than 5 edges. Polygons are named by the number of edges or angles they have - see table at bottom.

**vertex** A vertex is a point where two edges of a **polygon** meet to form a corner.

**interior vertex angle** An interior vertex angle is the angle formed INSIDE a **polygon** between two adjacent edges.

**exterior vertex angle** The size of an exterior angle at any vertex is  $180^\circ - \text{interior vertex angle}$  (it may be negative)

For ANY polygon the sum of all the exterior vertex angles is  $360^\circ$

**angle sum** The angle sum of a **polygon** is the total of ALL its **interior vertex angles** added together. For a triangle it is  $180^\circ$ ; for a quadrilateral  $360^\circ$

Angle sum of any polygon =  $(180 \times \text{number of edges}) - 360$  degrees

**equilateral** An equilateral **polygon** is one whose edges are ALL the same length.

**equiangular** An equiangular **polygon** is one whose **interior vertex angles** are ALL the same size.

**isogon**  $\equiv$  equiangular polygon

**regular** A regular **polygon** is one which is both **equilateral** and **equiangular**.

**concave** A concave **polygon** is one having at least one **interior vertex angle** which is greater than  $180^\circ$

**convex** A convex **polygon** is one whose **interior vertex angles** are ALL less than  $180^\circ$ . All regular polygons are convex.

**circumcircle** The circumcircle to a polygon is a circle around the OUTSIDE of a **polygon** passing through ALL its vertices. It is always possible to draw a circumcircle for any regular polygon, but may not be possible for an irregular polygon.

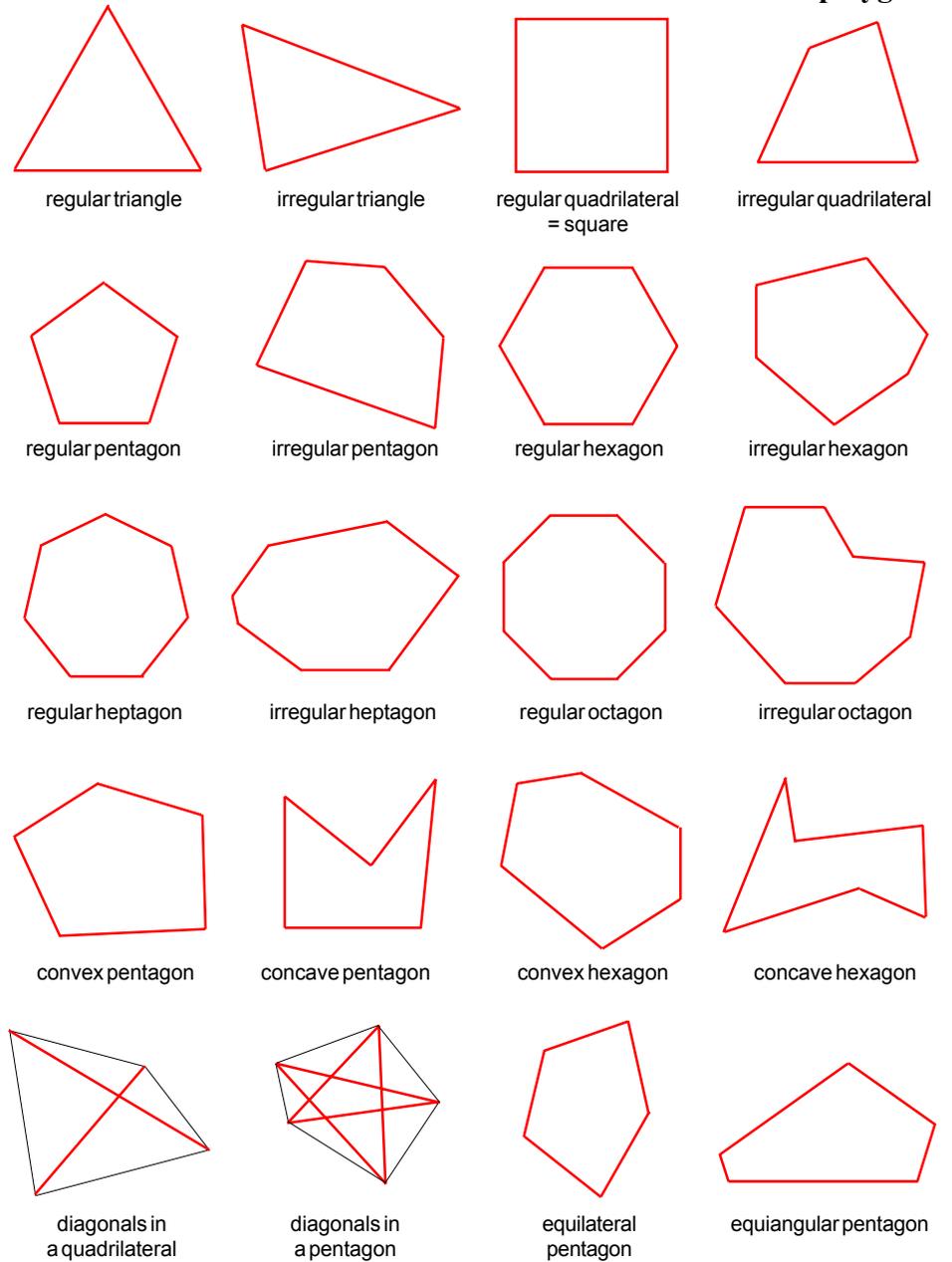
**incircle** An incircle to a polygon is a circle drawn INSIDE a **polygon** that touches ALL its edges. It is always possible to draw an incircle for any regular polygon.

Table of data for **regular polygons**

No. of edges	Name	Area = $e^2 \times \text{by}$	C-radius = $e \times \text{by}$	I-radius = $e \times \text{by}$	Int. vertex angle $^\circ$
3	triangle	0.43301	0.5774	0.2887	60
4	quadrilateral	1	0.7071	0.5	90
5	pentagon	1.72048	0.8507	0.6882	108
6	hexagon	2.59808	1	0.8660	120
7	heptagon	3.63391	1.1524	1.0383	128.57
8	octagon	4.82843	1.3066	1.2071	135
9	nonagon	6.18182	1.4619	1.3737	140
10	decagon	7.69421	1.6180	1.5388	144
11	undecagon	9.36564	1.7747	1.7028	147.27
12	dodecagon	11.1962	1.9319	1.8660	150

C-radius and I-radius refer to the circumcircle and incircle respectively.  $e$  is the length of one edge. Inexact values are given to 5/6 significant figures.

## polygons



## quadrilaterals

**quadrilateral** A quadrilateral is a **polygon** which has 4 edges. *Its 4 vertex angles add up to 360 degrees.*

**trapezium** A trapezium is a **quadrilateral** with only one pair of parallel edges.

**trapezoid**  $\equiv$  **trapezium** in N.American usage, but in UK usage it is a quadrilateral in which no two opposite edges are parallel.

**isosceles trapezium** An isosceles trapezium is a **trapezium** in which the two opposite edges, which are not parallel, are the same length. *It has one line of symmetry and both diagonals are the same length.*

**parallelogram** A parallelogram is a **quadrilateral** which has two pairs of parallel edges. *It has rotational symmetry of order 2, and its diagonals bisect each other. Usually one pair of edges is longer than the other pair, no vertex (=corner) angle is a right angle and it has no lines of symmetry.*

**rhombus** A rhombus is a **quadrilateral** whose edges are all the same length. *Its diagonals bisect each other at right angles and both are also lines of symmetry. Usually no vertex (=corner) angle is a right-angle and then it is sometimes referred to as a **diamond, lozenge, or rhomb.***

**rhomboid** A rhomboid is a **parallelogram** having adjacent edges of different lengths. *The word is little used because of possible confusion.*

The area of a trapezium, parallelogram or rhombus can be found by adding together the lengths of one pair of parallel edges, dividing by 2, and multiplying this by the perpendicular distance between them.

**rectangle** A rectangle is a **quadrilateral** in which every vertex (=corner) angle is a right angle.

**oblong** An oblong is a **rectangle** in which one pair of edges is longer than the other pair. *It has two lines of symmetry and rotational symmetry of order 2. Both diagonals are the same length and bisect each other.*

**square** A square is a **rectangle** whose edges are all the same length. *It has four lines of symmetry and rotational symmetry of order 4. Both diagonals are the same length and bisect each other at right angles.*

**kite** A kite is a **quadrilateral** which has two pairs of adjacent edges (= edges which are next to each other) of the same length, and no vertex angle (=corner) is bigger than 180 degrees. *It has one line of symmetry and its diagonals cross each other at right angles.*

**arrowhead** An arrowhead is a **quadrilateral** which has two pairs of adjacent edges of the same length and one vertex angle bigger than 180 degrees. *It has one line of symmetry and its diagonals do not cross. It is also known as a **dart or deltoid.***

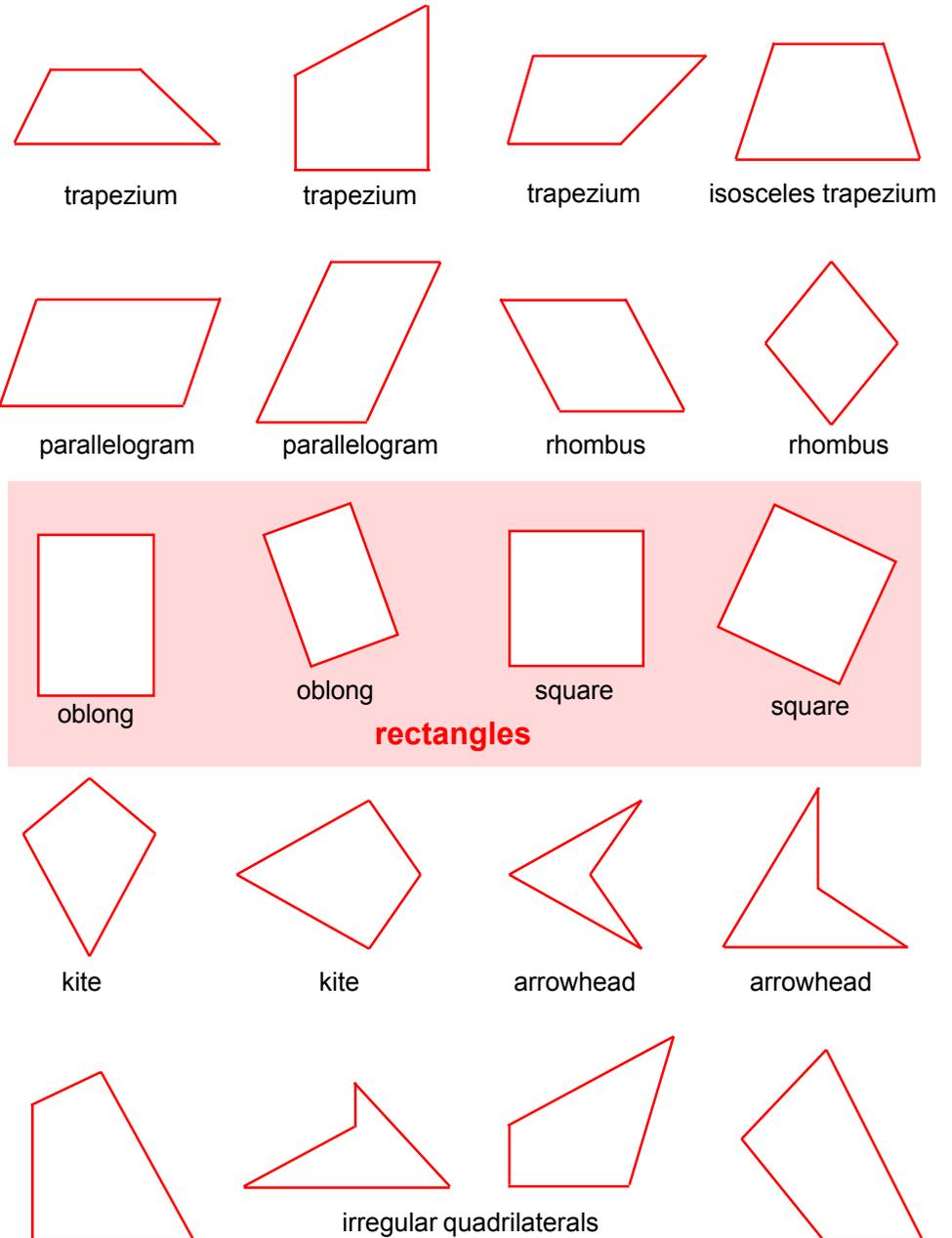
**regular quadrilateral**  $\equiv$  **square.**

**irregular quadrilateral** Strictly speaking, an irregular quadrilateral is any **quadrilateral** that is not a square, but it is usually taken to be one not having a special name.

**golden rectangle** A golden rectangle is an **oblong** with its two edge-lengths sized in the proportions of the golden ratio ( $\approx 1.618 : 1$ ).

Length of longer edge  $\approx 1.618 \times$  length of shorter edge.

## quadrilaterals



## recreational mathematics (general)

**recreational mathematics** covers games, puzzles and similar activities in which mathematical principles are used in some way: to create them, to play them or to solve them. The types and examples given here are just a sample drawn from the very wide range of material that is available.

**cryptarithms** are sums in arithmetic in which some (or all) of the original digits have been hidden in some way, and it is required to find them.

**alphametics** are **cryptarithms** in which all of the digits have been replaced by letters (each letter always representing the same digit, and each digit always represented by the same letter) and with the arrangement forming real words.

**asterithms** are **cryptarithms** in which some of the digits have been replaced by asterisks, and the puzzle is to find the correct values of the missing digits.

**four 4's** This is a particular type of problem which originated in 1871. It requires as many whole numbers as possible to be represented by the use of four 4's joined by mathematical operations. Some examples are shown. There are usually no rules or restrictions placed on the operations allowed, leaving it to the abilities and knowledge of the individual. From this beginning other problems have been posed. The 4 can be replaced by any other digit. The digits can be different from each other - in this case it may be specified whether they are to be kept in order or not. In recent years there has been a tendency to use the year itself, so for example, 1998 yields  $1 \times 9 + 9 - 8 = 10$ . Another variation is to ask for the largest (or smallest) possible number to be made using only the digits given. In all of this work it is particularly important to be aware of the **order of operations**.

**counting out problems** are based on the idea that a set of objects (or people) are arranged in a circle and then, after counting up to some number  $n$  while moving around the circle, the  $n$ th object is removed; this being continued until only one object is left. *It is often seen in children's games where one has to be chosen for a certain task, and the counting is done by using a rhyme. Problems are based on a requirement to predetermine where in the circle an object should be placed to ensure it is selected.*

*Example: 12 playing cards (Ace to King) are arranged in a circle and then, starting the count at the 5, every 7th card is removed until none are left. How must they be arranged so that the cards removed are in their correct numerical order, from the Ace to the King?*

**tricks** There are many tricks involving the manipulation of numbers (often using cards, dice and dominoes) which seem to produce 'magical' results, but which depend for their working only on elementary principles of arithmetic.

*Example: Ask someone to roll two dice without revealing the results. Then to multiply one of them by 5, add 7, double the result and add the other number, and give the result. From the two digit answer, subtract 14 and the two digits remaining will give the numbers showing on the dice.*

$$\begin{array}{r} \text{S E N D} \\ + \text{M O R E} \\ \hline \text{M O N E Y} \\ \hline * 4 6 \\ + 2 8 * \\ \hline 4 * 1 \end{array}$$

$$\begin{aligned} 1 &= (4 + 4) \div (4 + 4) \\ 2 &= (4 \div 4) + (4 \div 4) \\ 3 &= (4 + 4 + 4) \div 4 \\ 4 &= 4 + 4 - \sqrt{4} - \sqrt{4} \\ 15 &= 44 \div 4 + 4 \\ 17 &= 4 \times 4 + 4 \div 4 \\ 27 &= 4! + \sqrt{4} + (4 \div 4) \end{aligned}$$

## recreational mathematics (general)

**measuring problems** have always been popular because they are easy to understand, and do relate to something familiar in ordinary life, though some constraints have to be introduced in order to make it a problem. One of the earliest type was introduced by **Tartaglia**: "*There are three uncalibrated containers of 3, 5 and 8 litres capacity and the largest one is full. How can the 8 litres be divided into two lots of 4 litres?*" Some measuring problems are based on the use of weights, while others require lengths to be measured with imperfectly marked rulers.

**ferry problems** is another class of problems which are usually easy to understand but not so easy to resolve. The oldest (and easiest) of this type concerns the traveller with a wolf, a goat and a bag of cabbages who needs to cross a river using one boat and taking only one of the things at a time. But, the wolf and goat can never be left alone together, and nor can the goat and cabbages.

**Diophantine problems** seem at first sight to have insufficient information for their solution but, because the objects involved can only be counted using whole numbers (such as live animals) at least one solution can be found.

*Example: In the market, ducks cost £5 each, hens cost £1 each and baby chickens were 20 for £1. Kim bought at least one of each, a 100 birds altogether, for a total of £100. How many of each did Kim buy?*

**magic squares** A magic square is a set of numbers arranged in the form of a square so that the total of every **row**, **column** and **diagonal** is the same. *It is usual to require that every number is different. In most cases the numbers also form some kind of sequence. Examples:*

using 1 to 9	using 1 to 16	using primes only
8 3 4	16 2 3 13	17 89 71
1 5 9	5 11 10 8	113 59 5
6 7 2	9 7 6 12	47 29 101
	4 14 15 1	

**Latin squares** A Latin square of size  $n$  by  $n$  is one in which  $n$  different objects are each repeated  $n$  times and arranged in a square array so that none of them is repeated in any row or column.

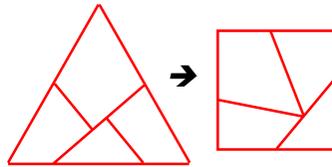
**Graeco-Latin squares** A Graeco-Latin square is made by combining two **Latin squares** (each of which is made of a different set of objects) so that no **PAIR** of objects is repeated. It is also known as an **Euler square**. *Squares of this type are useful in the design of experiments. Example:*

A B C D		1 2 3 4		A1 B2 C3 D4
C D A B		4 3 2 1	=	C4 D3 A2 B1
D C B A	+	2 1 4 3		D2 C1 B4 A3
B A D C		3 4 1 2		B3 A4 D1 C2

## recreational mathematics (spatial)

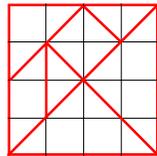
**dissections** Dissection puzzles require one shape to be cut up into a number of pieces and these pieces then re-assembled to make some other shape. In some cases the pieces are given and the puzzle is to find how they can be used to make another shape - which is usually specified.

Example: Dissect an equilateral triangle into the smallest possible number of pieces that can be rearranged to make a square. The solution is shown



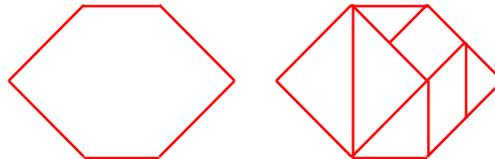
**tangram** Tangram puzzles use a standard dissection of a square into 7 pieces and require arrangements of those pieces to be found which will make up a given shape. The given shape is shown only in outline.

Making the pieces

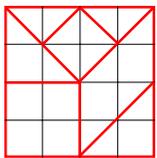


The grid is only for guidance. The red lines show how the square is to be cut up.

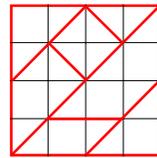
A tangram puzzle . . . . . and its solution



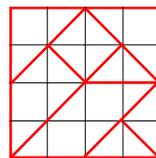
**polygrams** is the general name for those puzzles which are similar in type to tangrams, but which use a different dissection of the square, or even a different shape altogether. Below are some varieties that have been commercially produced in the past, with their names.



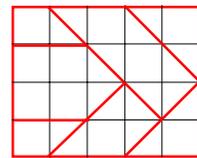
Pythagoras



Chie No Ita



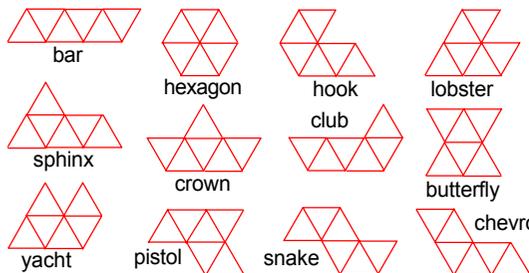
Tormentor



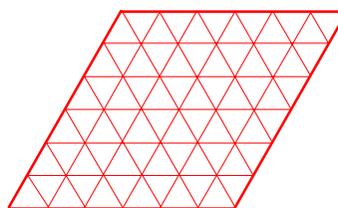
Cross Breaker

**polyiamonds** are 2-dimensional shapes made from identical equilateral triangles joined together by their edges. Each type is named from the number of triangles it uses: 4 triangles = tetriamond; 5 triangles = pentiamond etc.

**hexiamonds** are the 12 different polyiamonds that can be made from 6 triangles.



The 12 hexiamonds make a rhombus

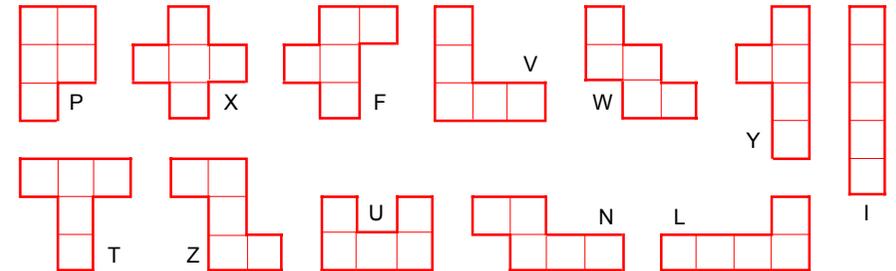


## recreational mathematics (spatial)

**polyominoes** are 2-dimensional shapes made from identical squares joined together by their edges. Each type takes its particular name from the number of squares it uses: 4 squares = tetromino; 6 squares = hexomino etc. The number of different shapes that can be made with any given number of squares can be worked out, but there is no formula to determine this relationship, which is given in the following table:

No. of squares	1	2	3	4	5	6	7	8	9	10
No. of shapes	1	1	2	5	12	35	108	369	1285	4655

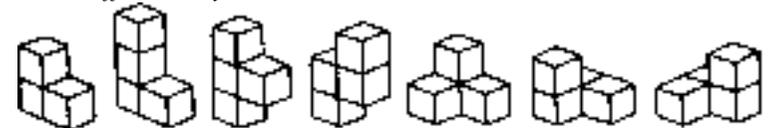
**pentominoes** are the 12 different polyominoes that can be made from 5 squares. Of all the polyominoes, these have proved to be the most popular set. The set is big enough to work with, but small enough to be handleable. Each piece is identified by a letter as shown below. One puzzle (out of many) for these pieces is to make a rectangle using all 12 of them.



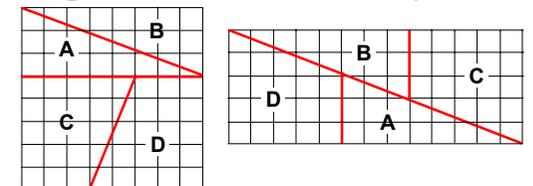
**polycubes** are 3-dimensional shapes made from identical cubes joined together by their faces. The number of different shapes that can be made with any given number of cubes is always of interest, but there is no formula to determine this relationship, which is given in the following table:

No. of cubes	1	2	3	4	5	6	7	8	9	10
No. of shapes	1	1	2	8	29	166	1023	6922	48311	346543

**Soma cubes** are the 7 different polycubes that can be made from 3 or 4 cubes, with each having at least one concave corner. It needs 27 cubes to make all seven. They can be assembled to make models of a wide variety of objects, and can be used to make a cube in 240 different ways. The 7 Soma acubes are shown here:



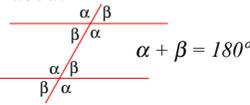
**illusions** An illusion is a visual trick showing something that cannot exist. The trick on the right shows an area of 64 squares cut up and re-arranged to make a 5 by 13 oblong with an area of 65 squares.



## symbols and their origins

**Greek alphabet** The Greek alphabet is a rich source of symbols used in mathematics and some of them appear more than once to represent different things. Below is the full alphabet, with a note of where some of them are used.

A  $\alpha$  alpha  $\alpha, \beta, \gamma$  are often used to identify angles  
 B  $\beta$  beta  
 Γ  $\gamma$  gamma  
 Δ  $\delta$  delta  $\Delta$  is sometimes used to represent the area of a **triangle**.  
 $\delta$  is used to show that a small measure is to be taken.  
 Example:  $\delta y$  would mean 'a small amount of  $y$ '.



E  $\epsilon$  epsilon  
 Z  $\zeta$  zeta  
 H  $\eta$  eta  
 Θ  $\theta$  theta  $\theta$  is used to indicate a general angle as in **polar coordinates**.  
 I  $\iota$  iota  
 K  $\kappa$  kappa  
 Λ  $\lambda$  lambda  $\lambda$  is used to represent a **scalar** in vector work.  
 M  $\mu$  mu  $\mu$  is used in the **SI** system of units to represent the prefix *micro*.  
 Example:  $\mu m$  is a micrometre, or one-millionth of a metre.  
 $\mu$  is sometimes used to represent the **arithmetic mean**.

N  $\nu$  nu  
 Ξ  $\xi$  xi  $\xi$  is sometimes used as the symbol for the **universal set**.  
 O  $\omicron$  omicron  
 Π  $\pi$  pi  $\Pi$  is used to show that a **continued product** is needed.  
 $\pi$  is used to represent the **irrational number** 3.14159...  
 $\pi(n)$  means the number of primes equal to or less than  $n$ .  
 Example:  $\pi(13)$  is 6. The six primes are 2, 3, 5, 7, 11, 13

$n$	$\pi(n)$	$n$	$\pi(n)$	$n$	$\pi(n)$
10	4	10 000	1229	10 million	664 579
100	25	100 000	9593	100 million	5 761 455
1000	168	1 million	78 499	1 billion	50 847 534

P  $\rho$  rho  
 Σ  $\sigma$  sigma  $\Sigma$  is used to show that the sum of a **series** is to be found.  
 $\sigma$  is used to represent the **standard deviation** of a population.  
 $\tau$  is used to represent the value of the **golden ratio** 1.6180...  
 T  $\tau$  tau  
 Y  $\upsilon$  upsilon  
 Φ  $\phi$  phi  $\Phi$  is sometimes used (incorrectly) to represent an **empty set**  
 X  $\chi$  chi  
 Ψ  $\psi$  psi  
 Ω  $\omega$  omega

## symbols and their origins

- + - One of the earliest signs used to show that two numbers had to be added was the Ancient Egyptian hieroglyph which represented a pair of legs walking forward (their writing and reading was done from right to left).  
 So would have been read as 3 + 12. Unsurprisingly, their minus sign was a pair of legs walking in the opposite direction. Up until the 1500's a variety of signs were used, but very often the instruction was written in full. Italian mathematicians of the 1400's used  $p$  and  $m$  (for *plus* and *minus*) which was a shortened form of their (Italian) words. The first + and - signs appeared in 1481 in a German manuscript on algebra. For quite some time their use appears to have been restricted only to algebra and it took nearly 100 years before they came into more general use in arithmetic.
- × As with the + and - signs, there was a wide diversity of symbols used for multiplication. Many writers used lines to show which numbers were to be connected in some way and, inevitably, some of these lines crossed, so while the × appeared in many places it did not always mean that multiplication had to be done. It was the English mathematician William Oughtred who (in 1631) gave it the particular meaning it has today. Some objected it was too much like an  $x$ , but, nevertheless, it was slowly adopted over the next century.
- ÷ This was once commonly used as a minus sign. Then a Swiss mathematician, Johann Rahn, used it to show division in 1659 and it was adopted in England and the USA, but Continental Europe continues to use the colon : (as introduced by **Leibniz**) to show division.
- = The sign for equality we use today was introduced by **Recordes** in his book *The Whetstone of Witte (1577)* where he justified it by explaining "a paire of paralleles, lines of one lengthe, thus: , bicause noe.2.thynges, can be moare equalle." It was not immediately adopted by everyone. As with much mathematical notation of that period, everyone had his (very rarely her) own system, but by about 1700 the = sign was in universal use.
- √ Early writers (pre-1500) used a dot to show that a root was to be taken. (The dot as a decimal point came later). It can only be conjectured that a 'tail' was added to the dot to make it more visible, because that is what some writers started to do. The present form of the sign was introduced in 1525 and was gradually taken into use over the next 100 years or so.
- **decimal fractions** were known about from very early times, though they were not used a lot even where they would have simplified the calculation. Their regular use was helped when Simon Stevin published (in 1585) a very clear description of them. However, he used a clumsy notation and it was **Napier** (in 1616) who introduced a decimal separator (comma or point) which made decimal fractions much easier to use.
- ! The **factorial** sign, like most mathematical symbols, has been represented in many different ways. Some of the simpler ones were  $n^*$   $\Pi(n)$   $[n]$   $\underline{n}$  all representing what we now write as  $n!$  The use of the exclamation mark was introduced in Germany in the early 1800's, but it was not until the middle of the 1900's that it could be said to be in universal use.

As can be seen from the previous pages, the dictionary is set out in a series of double-page spreads, with each spread devoted to a particular theme.

In this way, cross-references (printed in **bold type**) are, in most cases, to be found on the same page.

It also means that words which are usually found within the same context are to be found together in the dictionary and serendipitous browsing is encouraged.

There is a Wordfinder in the front of the dictionary to enable individual words to be found.

Below are listed the 84 themes used to make the double-page spreads.

**abbreviations and mnemonics**  
**accuracy**  
**algebra (basics)**  
**algebra (equations)**  
**algebra (functions)**  
**angles**  
**arithmetic (basics)**  
**arithmetic (commercial)**  
**arithmetic (social)**  
**arithmetic (the four rules)**  
**calculating aids**  
**calculus**  
**circle**  
**circles and their properties**  
**circles related to other shapes**  
**cones, cylinders and spheres**  
**conic sections**  
**coordinate systems**  
**curves**  
**cycloids**  
**eponyms**

**factors, multiples and primes**  
**famous problems**  
**formulas for shapes**  
**fractions**  
**geometry**  
**graphs**  
**information technology**  
**instruments**  
**instruments & mechanisms**  
**kinematics**  
**logic (language)**  
**logic (practice)**  
**mathematicians of earlier times**  
**mathematicians of later times**  
**matrices**  
**navigation**  
**number diversions**  
**number forms**  
**number systems**  
**numbers**  
**patterns**

The first edition of this dictionary was also published (in the USA) by Barron's as

***Barron's Mathematics Study Dictionary***

ISBN 0-7641-0303-2

That version does NOT contain some of the pages shown here.

**pi ( $\pi$ )**  
**polygon numbers**  
**polygons**  
**polyhedrons**  
**probability**  
**pyramids and prisms**  
**quadrilaterals**  
**recreational mathematics (1)**  
**recreational mathematics (2)**  
**sequences and series**  
**sets**  
**shapes**  
**space and shapes**  
**statistics (basic)**  
**statistics (general)**  
**statistics (graphical)**  
**statistics (numerical)**  
**statistics (techniques)**  
**surveying**  
**symbols**  
**symbols and their origins**

**symmetry**  
**technical drawing**  
**techniques**  
**temperature**  
**tessellations**  
**topology**  
**transformation geometry**  
**triangles**  
**trigonometry (basic)**  
**trigonometry (further)**  
**units and conversions**  
**units and the SI**  
**vectors (graphical)**  
**vectors (numerical)**  
**word confusions**  
**word origins**  
**words from further mathematics**  
**words in general use**  
**words of wider interest**  
**words with their pronunciation**  
**words in other languages**