Circles

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The Circle

Relevant formulas are:

- Circumference of circle = \( \pi \times \text{diameter} \) or \( C = \pi d \)
- Area of Circle = \( \pi \times \text{radius} \times \text{radius} \) or \( A = \pi r^2 \)

Use a value for \( \pi \) of 3.14

Remember to show all working, give answers to the degree of accuracy asked for, and attach units to the answer. Questions marked * have numerical answers printed at the bottom of the sheet. They are not in order, and will have to be found amongst all the numbers given.

Section A (Give answers to 2 decimal places)

* 1. Find the circumference of a circle having a diameter of 8 cm.
* 2. What is the circumference of a circle with a radius of 7.5 metres?
* 3. Find the area of a circle with a radius of 4 cm.
* 4. Calculate the area of a circle whose diameter is 5 metres
* 5. Find the circumference of a circle whose radius is 2.6 metres.
* 6. What is the area of a circle having a radius of 1.7 cm?
* 7. Calculate the circumference of a circle having a diameter of 4.7 metres.
* 8. A circle has a diameter of 6.3 cm. What is its area?

Section B (Give answers to 3 significant figures)

* 9. A circle has a circumference of 40 cm. Find its diameter.
* 10. What is the radius of a circle which has a circumference of 5 metres?
* 11. Calculate the radius of a circle which has an area of 15 square metres.
* 12. Find the diameter of a circle whose area is 70 square metres.
* 13. Calculate the diameter of a circle having a circumference of 12.7 metres.
* 14. A circle has an area of 23.8 m². What is its radius?

Section C (Give answers to an appropriate degree of accuracy)

* 15. A bicycle-wheel has a diameter of 75 cm. What is the circumference of the wheel?
* 16. The minute hand of one clock is 12.7 cm long. How far does the tip of the hand travel in 1 hour?
  (a) How far will the barrow have travelled when the wheel has turned just once?
  (b) How many metres will the barrow go in turning the wheel 10 times?
* 17. The single wheel of a wheel-barrow has a diameter of 47 cm.
  (a) How far does a cyclist travel in one lap while keeping close to the inner edge?
  (b) How much further does he travel by keeping 1 metre out from the inner edge?
* 18. The inner edge of an indoor racing cycle track is a circle of diameter 127.3 metres.
  (a) How far does a cyclist travel in one lap while keeping close to the inner edge?
  (b) How much further does he travel by keeping 1 metre out from the inner edge?
* 19. A wheel has a diameter of 80 cm. Calculate its circumference, and then find how many complete turns it must have made in moving forwards 100 metres.
* 20. A donkey is tethered in the middle of a field on a rope 9 metres long. Find the area of grass over which the donkey can graze.
* 21. A circular clock-face has a diameter of 16.5 cm. Calculate its area.
* 22. The average tea-plate has a diameter of 15 cm and the average dinner-plate a diameter of 25 cm. Find the areas of these plates.
* 23. Two circles have radii of 3.5 cm and 7.0 cm respectively. How many times bigger in area is the larger circle that the smaller circle?
* 24. Repeat the previous question for two circles of radii 2 cm and 6 cm.
* 25. A piece of thread is wrapped 5 times completely around a circular tin. When unwrapped and laid out straight the string measures 108 cm. What is the diameter of the tin?

The numerical answers (no units given) to questions marked * will be found amongst these

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Drawing Circles

The radius of a circle is the distance from its centre to the edge of the circle. This is the measurement needed to draw the circle with a pair of compasses.

The diameter of a circle is the length of a straight line drawn across the circle from edge to edge, and passing through the centre. Its length is twice that of the radius.

The circumference of a circle is the distance all the way around the edge of the circle. In any other shape it would be called the perimeter.

Relevant formulas are:

- Circumference of circle = \( \pi \times \text{diameter} \)
- Area of Circle = \( \pi \times \text{radius} \times \text{radius} \)

Use a value for \( \pi \) of 3.14

In each of these examples, draw the circle(s) asked for, mark the position of the centre, write the information given beside it and, if it was not given, write the radius used to draw it.

As a guide to spacing, Section A can be fitted on one side of a sheet of A4, and Section B on the other.

Example: Draw a circle whose area is 9.6 cm\(^2\).

Section A

1. Draw a circle using a radius of 2.6 cm.
2. Draw a circle with a circumference of 12.6 cm.
3. Draw a circle having an area of 43 square cm.
4. Draw a circle which has a circumference of 26.4 cm.
5. Draw a circle which has an area of 32.2 cm\(^2\).
6. Draw a circle with a circumference of 24.5 cm.
7. Draw a circle whose diameter is 5.6 cm.

Section B

In each of these, 2 circles have to be drawn on the same centre.

8. One to have an area of 56 cm\(^2\), the other to have an area of 20 cm\(^2\).
9. One with a circumference of 22 cm, the other with a circumference of 16 cm.
10. One having an area of 17 cm\(^2\), the other having a circumference of 17 cm.
11. The area of one to be 8.4 cm\(^2\), while the circumference of the other is 8.4 cm.
12. One to have a circumference of 24 cm, the other to have half that circumference.
13. One with an area of 22 cm\(^2\), the other being twice that in area.
The Annulus

An **annulus** is the shape formed between two circles, one being completely contained inside the other. Usually both circles have a common centre.

Here, \( d, r, a \) refer to the diameter, radius and area respectively of the **inside** circle. with \( D, R, A \) being used for the **outside** circle.

\( w \) is the width of the annulus. It is the difference between the two radii. \( w = R - r \)

Complete the table below.
Sizes are given only as numbers. Assume units are compatible throughout.
Use a value for \( \pi \) of 3.14 and give answers to 1 decimal place.

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</table>
Circles 2

1. A dartboard has an overall diameter of 41 cm. Calculate the total area in which the dart may land. (Note that not all of this is the scoring area.)

2. A circular table of diameter 2.5 metres is to have a plastic strip fixed around its outer edge. Find the length of plastic strip needed.

3. A flat metal washer has inside and outside diameters of 1.3 and 2.5 cm respectively. What is the area of the washer?

4. A circular pond is 17.5 metres in diameter. It is surrounded by a path which is 2 metres wide. What is the area covered by the path?

5. The diameter of one car wheel (over the tyre) is 63 cm.
   (a) How far will the car have moved forward when the wheel has turned once?
   (b) How many times will the wheel have turned when the car has moved 100 metres?

6. The drawing on the right shows a basic archery target marked with its five scoring areas, identified from the centre outwards as Gold, Red, Blue, Black and White. The scores for each of these are 9, 7, 5, 3 and 1 respectively. The Gold is 25 cm in diameter and each of the other bands is 12.5 cm wide. Find the actual areas of each of the five scoring areas.

7. A square of sheet-metal has an edge-length of 150 cm. What is the area of the largest possible circle that can be cut from this square, and what area of the square will remain?

8. From a large circle of diameter 17 cm, two smaller circles each having the same diameter as each other, are cut out. If these two circles are to be as big as possible, what area of the original circle will be left?

9. A circular mirror of diameter 35 cm is set in a gilded surround which is 9 cm wide. Calculate the area of gold-leaf needed to gild the surround.

10. Water is raised from a well by means of a rope being wound up around a drum. The well is 20 metres deep and the drum has a diameter of 30 cm. How many times will the drum be turned in raising one bucketful of water?

11. A piece of string 300 cm long can be wrapped around a circular tin exactly 8 times. What is the diameter of the tin?

12. A circular fish-pond is to be made to take 40 fish. Good practice requires that for each fish there should be half a square metre of water surface in contact with the air. What should be the minimum diameter of the pond?

13. A car travels 1 kilometre and, in that distance, each wheel makes 450 complete turns. What is the diameter of its wheels?

14. A car has wheels which are 80 cm in diameter. How many complete revolutions does each wheel make in 1 minute when the car is travelling at
   (a) 80 km per hour
   (b) 80 miles per hour?
**Sectors**

A **sector** of a circle is the shape formed or enclosed by an arc and the two radii running from the centre of the circle to each end of that arc.

Relevant formulas are:

\[
\text{Arc length} = \frac{A^\circ}{360} \times \text{Circumference of complete circle}
\]

\[
\text{Area of sector} = \frac{A^\circ}{360} \times \text{Area of complete circle}
\]

Note that whenever one sector is created from a circle then the piece of the circle remaining is also a sector. If necessary to distinguish between them, the larger one is known as the **major sector** (\(A^\circ\) is greater than 180°) and the smaller one is the **minor sector** (\(A^\circ\) is less than 180°). When \(A^\circ = 180^\circ\) then the sector is a **semicircle** and the two sectors are equal in size.

Using a value for \(\pi\) of 3.14 complete the table below, giving answers to 1 decimal place.

<table>
<thead>
<tr>
<th>For complete circle</th>
<th>For a sector of that circle</th>
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</thead>
<tbody>
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<td>22.</td>
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</tbody>
</table>
1. A circle has an area of 400 cm$^2$. What is its circumference?

2. A circle has a circumference of 100 cm. What is its area?

3. A flower-bed is in the shape of a semicircle of diameter 6.5 metres.
   (a) What is the length of the perimeter of this flower-bed?
   (b) What is its area?

4. A goat is tethered to one corner of a rectangular field. The rope by which it is tethered is 7.5 metres long. Calculate the area of the field over which the goat can graze.

5. The largest tree in the world (known as General Sherman and in California) has a girth (= circumference) of 25.3 metres. Calculate its diameter.

6. On a dartboard, the inner and outer diameters of the trebles ring are 19.05 and 20.95 cm respectively. The corresponding figures for the doubles ring are 31.75 and 33.65 cm. Calculate the areas of each of these two rings.

7. An umbrella has 8 ribs which are equally spaced. Assuming the umbrella to be a flat circle of diameter 90 cm, what is the area of 1 panel (between 2 ribs) of the umbrella?

8. To warn shipping of underwater rocks, a lighthouse shines a red-coloured light over a 95° sector out to a distance 16.5 km. Over what area of sea are ships being warned to keep clear?

9. The radius of the Earth is approximately 6400 kilometres. A satellite is orbiting the Earth at a height of 200 km above the Earth's surface. How far does the satellite travel in one complete orbit?

10. A drain-cover is made from a square metal plate of edge length 40 cm, by having 676 holes drilled in it, each of 1 cm diameter. When the cover is complete, what area of the original square plate remains? (Why 676 holes?)

11. A cart-wheel is bound around its rim with a band of steel which is 3.75 metres long. What is the diameter of the wheel? Another cart-wheel is 50 cm bigger in diameter. How much longer will the steel band for the rim of that wheel have to be?

12. A car has two windscreen wipers which do not overlap. Each has a blade of 25 cm length sweeping through an angle of 115°. Calculate the area of windscreen cleaned at each sweep of the blades.

13. Four equal sized tins of diameter 9.5 cm are stood together 2 by 2. To hold them together, a piece of string is wound around them just once. The knot to tie the string needs 11 cm. How much string is needed in total?

14. A circle has a diameter of 9.3 cm. A sector representing 2/5ths of the circle is cut out.
   (a) What is the angle of this sector?
   (b) What is the arc length?
   (c) What area of the circle remains?

15. The minute-hand of the largest clock in the world is 6 metres long. Calculate how far the tip of the minute hand travels during
   (a) 1 hour
   (b) 30 minutes
   (c) 5 minutes
   (d) 1 minute
   (e) 23 minutes
   (f) 30 seconds.

16. The sketch on the right is of a brooch which is to be made in silver wire. The circle will be 35 mm in diameter. Calculate the total length of silver wire required.