Division

The generally accepted hierarchy of difficulty for the 4 basic operations of arithmetic is (from easiest to hardest): addition, subtraction, multiplication, division. When it is noted how heavily the last is dependent upon the previous two, it is clear why so many (most?) pupils leave school unable to do division without the aid of a calculator.

This unit is not concerned with the actual introduction and teaching of how a division algorithm is carried out but only with providing working material which is structured in a way that progresses very gradually from the very easiest to the quite difficult.

Three assumptions that are made are:

- that pupils are learning to use a 'traditional' division algorithm;
- that pupils do not know their tables, but can use a multiplication table when it is provided;
- that pupils can do subtraction with a good degree of accuracy.

Background

The symbols ÷ and / are the main ones used to indicate division, so $6 \div 3$ and $6 / 3$ mean the same thing.

÷ is used in written and printed work, and also on a calculator.

/ is used in computing.

The symbol – as in $\frac{6}{3}$ can be included since that is its ultimate meaning in that situation.

The symbol **:** was once used to mean division (and ÷ meant subtraction) but in the mid-1600's English mathematics adopted the symbol we know and use today*.
Continental Europe retained (and still uses) **:** to mean division.
English mathematics retained **:** in one particular place; using it to show ratio as in $6 : 3$ which can be considered as a form of division.

In saying it there are many variations such as (all meaning $6 \div 3$)

"six divided by three"
"six shared by three"
"threes into six"
"how many threes in six?"

and every teacher can add to that list.
The first two are 'better' than the second two since the order of the numbers is maintained and this may go some way to obviating one of the common errors found in later work. But language in the classroom is always a matter of balance between 'correctness' and 'understanding'. No, they are are not mutually exclusive, but too much emphasis on one can result in some loss of the other. Teachers have to be negotiators also.

A commonly-found error is when a pupil has decided that division has to be done and then proceeds to divide the larger number by the smaller regardless of what was actually required. (The same is seen in subtraction.) One has every sympathy with this interpretation, because when you consider how the topic is introduced, and what most (all?) of the practice consists of, it is easy to see how it arises. But that does not make it right.
So, one important point that needs to made at some time in any pupil's arithmetic development is that division is NOT commutative.

Addition IS commutative:	$6 + 3 = 3 + 6$	order does NOT matter
Multiplication IS commutative:	$6 \times 3 = 3 \times 6$	order does NOT matter
Subtraction is NOT commutative:	$6 - 3 \neq 3 - 6$	order DOES matter
Division is NOT commutative:	$6 \div 3 \neq 3 \div 6$	order DOES matter

Clearly this is not something to bother about in the beginning, or it will be lost as just one message among many others (information overload) but it must come at some time.

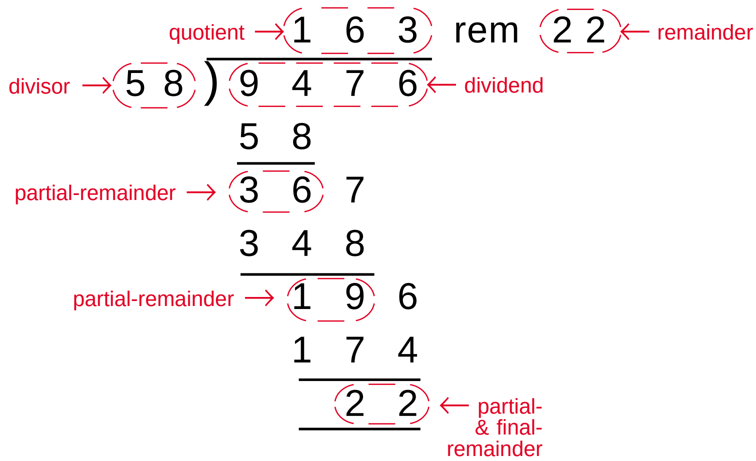
* The symbol ÷ is known as an **obelus**
It is a very old symbol but was not used to indicate division until the Swiss mathematician Johann Rahn first did so in 1659.

The naming of the parts

In order to detail the structure of the work given on the various sheets, it is necessary to make clear the meanings of the different technical terms used. This is most easily done by labelling all the parts of a typical 'long division' sum. Here the sum being done is

9476 ÷ 58   (dividend ÷ divisor)

Notice in particular the partial-remainders which are the results of the subtraction sums being done at various stages. These partial-remainders, together with the subtraction sum leading up to them, are important in making an assessment of the overall difficulty of any particular division sum.



Unfortunately, the word 'quotient' has two meanings which overlap.

1. The quotient is the result given by the operation, or process, of division.
In the example above, the quotient is: ***:*** *163 rem 22*

2. The quotient is the whole number part of the result given by the operation, or process, of division.
In the example above, the quotient is: *163*
(Notice that implicit in definition 2 is the fact that the remainder is not really a whole number but, more properly, is a fractional part. This is seen more clearly if the division is continued after inserting a decimal point in the answer line.)

Of course, these two meanings merge if there is NO remainder.
It is suggested that, at the elementary level, the word 'quotient' is not used, but only the word 'answer' which conforms to 'quotient 1', and removes the possibility of ambiguity. Should there be a need to refer to the ‘whole number part’ of the answer, then it could be named in just that way.
The word 'quotient' will not be used in these notes.

The choice of words to be used with pupils must be a matter of discretion and totally dependent upon the environment. For example, 'divisor' is a good short word but, 'the number you are dividing by' is much more user-friendly.

The term ‘partial-remainder’, as identified above, is introduced here only as a help in dealing with the analysis of the division algorithm in use.

Varieties of Division

‘Short’ or ‘Long’?

The terms ‘short division’ and ‘long division’ are often used. This unnecessary and can be confusing. Leaving aside peripheral distractions of historical methods (the Italian is a favourite) only one basic algorithm is taught in general. How that is handled varies. If the appropriate multiplication table is known and the intermediate subtractions can be done mentally, then the only writing that needs to be done is to record the answer. (Whether the partial-remainders are written in or not is irrelevant.) So the division sum 87934 ÷ 7could look like this

$$
\begin{array}{rccccc}
 & 1 & 2 & 5 & 6 & 2 \\ \cline{2-6}
7\,) & \mathbf{8} & {}^{1}\mathbf{7} & {}^{3}\mathbf{9} & {}^{4}\mathbf{3} & {}^{1}\mathbf{4}
\end{array}
$$

and it is not unreasonable to class this as ‘short division’.

It is in contrast to 9746 ÷ 58 (done on the previous page) which is clearly in the class of ‘long division’. However, it must be seen that the algorithm driving both of these sums is the same. It is only in its implementation that the difference arises, and this is entirely dependent upon the knowledge and skills of the user. So the shift from doing ‘short division’ to doing ‘long division’ can vary considerably between individuals. To accommodate that it is best if only the word ‘division’ is used and the techniques of application allowed to arise in their practice.

Types of Division

Dividing one number by another number is simply 'division'.

However, if named quantities are involved then two types of division are recognised.

Partition A partition is a division in which both the dividend and divisor have different names.

Quotition A quotition is a division in which the dividend and divisor have the same name.

Examples

*20 ÷ 5 is a **division***

*Dividing (or sharing) 20 apples among 5 people is a **partition**.*

(20 apples ÷ 5 people) or (apples ÷ people)

*Finding out how many 5 cm lengths can be cut from a piece of string 20 cm long is a **quotition**.*

(20 cm ÷ 5 cm) or (cm ÷ cm) or (length ÷ length)

Does it matter?

For teachers it certainly ought to be known, because it has been observed that when division problems are set in a context, which almost inevitably means using named quantities, quotitions are less easily resolved than partitions - even though the numbers may be the same. In other words, it is not the division process itself which causes the problem, but in first deciding that division is appropriate, and then deciding which has to be divided by what.

(For anecdotes concerning this see Appendix 1)

Division

The Difficulties

The difficulties in the actual doing of a division sum, even when the algorithm has been properly mastered, arise in two principal areas.

The first is the demands upon knowing, or being able to generate, the appropriate multiplication table, or else having the ability to estimate and try out various multiples of the divisor. To overcome this and encourage practice in the use of the algorithm itself, the work set out in this unit has all of the divisors limited to two digits, and all the multiplication tables from 4 to 99 are supplied. The two master-sheets provided can be copied back-to-back on a single sheet, and there is a wide inner margin so that the sheet can be fitted into a binder. Each pupil should have his or her own sheet. These tables are also available in the form of a small booklet (which has to be made up) and can be found under 'Multiplication Methods' in the *trol* menu

The second difficulty is the demand for the many subtractions that have to be carried out. There is no easy fix for this, but it must be borne in mind, both in designing the work and in analysing errors. A hierarchy of sums can be identified ranging from the 'very easy' to the 'quite hard'. Some examples of this, are shown below. (Each 3 by 2 block represents a particular subtraction sum.)

2 5 9	8 7 6	4 8 3	3 5 8	6 4 3	5 7 4	4 0 7
1 3 4	8 6 9	4 5 6	2 7 4	1 7 5	4 8 9	2 6 8

They are in order of increasing difficulty from left to right. The first is straightforward. The second would need a little extra work if done formally, whichever method is used** but can be done easily by 'counting on'. Now it is not possible to produce many division sums which are limited to having only those two types of subtraction, besides being very unreal in practical terms. So, a few to get started maybe, but most of the work must involve subtractions of increasing difficulty. The message is clear: no pupil should be doing this work who does not have some competence and confidence in subtraction. Note that the partial-remainders can only be used as a first rough guide to assessing the probable order of difficulty of any division sum, but must be used with discretion. Consider,

$$\begin{array}{r} 579 \\ -\,123 \\ \hline 456 \end{array} \qquad\qquad \begin{array}{r} 903 \\ -\,347 \\ \hline 456 \end{array}$$

both produce the same answer (the partial-remainder) but the second is much harder that the first

** In formal subtraction sums there are two methods generally in use. One is known as **equal addition** (this is the one where words like 'borrow' and 'payback' arise) and the other is that of **decomposition**.

A common mistake is the '**missing zero syndrome**'. This arises when, during the course of doing a division sum a point comes where the divisor is smaller that the number that is currently being attempted. To understand how this particular mistake comes about, try the sum

61153154 ÷ 12

and see how the (wrong) answer 59697 is produced.

Introductory Work

There is a miscellaneous set of tables, printed in a larger type-face. These can be used to make an ohp slide. With this on view, perhaps exposing only the table needed at the time, various examples can be worked through on the board. A possible introductory sequence is outlined below. Only a few examples are given here to illustrate various points. In practice more would be needed, but they are easy to generate, and the small (single-digit) divisors guarantee 'easy' subtractions even if only 'counting-on' is available.

For the earliest work all the dividends could be of sufficient length to help build up the 'rythym' of the algorithm but, with small divisors so all the partial-remainders can be found mentally and, in the beginning, none are zero. *A slightly different approach would be to keep the dividends short so that individual sums do not seem tedious and mistakes are more quickly resolved. That would require a different set of examples from those given here, though the stages to be covered would be the same.*

589144 ÷ 4 7869285 ÷ 5

Make sure some examples contain a zero or two in the dividend.

6507792 ÷ 4 940704 ÷ 6

Then there is the case where the divisor is bigger than the leading digit of the dividend as in

436974 ÷ 6 246883 ÷ 7

Before doing those a decision must be made. Is a zero to be written (above the 4 & 2 respectively) or not? An argument for doing so is that this keeps the rules consistent throughout, that there is no need to treat the leading digits of the dividend differently from the rest and this might go some way towards reducing the 'missing zero syndrome'.

And then turn to those definitely needing a zero in the answer. (With small divisors this means the previous partial-remainder was zero.)

916104 ÷ 7 837648 ÷ 8

There is also a need to have some examples which leave a remainder **
The easiest way of generating these is to take a previous example with a known structure and add on something (less than the divider) to the last digit of the dividend. Of course, if all the fore-going has been absorbed, there should be no need to take any particular care in the construction at this point. Here is one which embodies 'everything'.

474358305 ÷ 9

** Sums requiring a remainder to be found have a useful function in 'defeating' those who will attempt to use a calculator. Very few are capable of working out the remainder on a calculator without some help.
Try asking for the remainder to 84 ÷ 37 and expect the answer 27 (from someone using a calculator)

An alternative is to give sums where the dividend is more than eight digits long. Most calculators cannot handle that.

Harder Examples

The move is now made into using bigger divisors.

A decision has to be made at some point concerning the presentation of the algorithm.
When is it to make an appearance in its traditional form? This would be as good a place as any at which to do it. Remember though that it is only a very handy way of organising the work. It could all be done by writing each subtraction sum down on one side as it was needed. Some people work very well like that.
For instance, the sum presented in its full traditional splendour on page 3, under the ‘The naming of the parts’ could well look like this

$$\begin{array}{r|l} & \ \ 1\ \ \ 6\ \ \ 3 \quad \text{rem } 22 \\ \hline 58 & 9\ \ {}^{0}4\ \ {}^{36}7\ \ {}^{19}6 \end{array}$$

with three small subtraction sums written elsewhere.

This may be undesirable for several reasons, but it is still a valid solution and still using the basic algorithm. The method of application must not be confused with the basic principle of the algorithm.

Assuming the traditional layout is to be used, many will benefit considerably if squared-paper is used as a guide to lining-up the digits.

The first stage is to show that the algorithm is still the same even though the partial-remainders start getting bigger. In the first example below, no partial-remainder is greater than 11. In the second example the biggest partial-remainder is 16 (at the end) and all the subtractions can be classed as easy.

88934762 ÷ 14 27957581 ÷ 23

After that it is a matter of gradually increasing the degree of difficulty of the subtractions to be done. These examples are structured to that end.

79989726 ÷ 34

98618986 ÷ 37

73797128 ÷ 44

78188409 ÷ 53

159864056 ÷ 68

655169073 ÷ 87

595910880 ÷ 96

A guide to the Practice Sheets

There are two Practice Sheets containing six Sections, identified as A to F. Each section contains forty division sums

Each of these sections has been wrritten in two parts. One part contains all the odd-numbered sums, the other contains all the even-numbered sums. As far as possible it has been done to produce two parallel sets of questions having the same order of difficulty overall. This allows the work to be assigned in such a way as to minimise the opportunities for collaboration. Assuming pupils are sitting in pairs, then the simple instruction

"Those on the left (nearest the window, or whatever) do the odd-numbered sums, those on the right do the even-numbered sums"

almost guarantees individual work.

It can be useful to inform pupils when remainders are not expected in the final answer, to serve as an indicator as to whether an answer might be correct or not. Whether to do this or not is a local decision. The information is not given on the worksheet.

Section A
Divisors are all single digit (4 to 9) and there are NO remainders throughout this section. Subtractions are of increasing difficulty, within the limits of such small divisors.

Section B
As for Section A except that now MOST (but not quite all) have remainders. Variation here would be to ask only for the value of the remainder to be stated. *(There are occasions in mathematics when we do want to know only what the remainder is.)*

Section C
All divisors lie in the range 11 to 51 (excluding multiples of 10) and there are NO remainders. Starting with the easiest, the subtraction sums increase in difficulty as progress is made through the section. There is one clear-cut distinction: in the first half nearly all the partial-remainders are single-digit; while in the second half they are nearly all two-digit.

Section D
This has the same overall structure as for Section C, except that now MOST (but not quite all) have remainders.

Section E
All divisors lie in the range 52 to 99 (excluding multiples of 10) and there are NO remainders. The subtractions are generally much 'fiercer' and nearly all of the partial-remainders are of the two-digit variety.

Section F
This has the same overall structure as for Section E, except that now ALL the questions generate remainders.

Division Practice ~ 1

Section A

1. 765 ÷ 5
2. 845 ÷ 5
3. 528 ÷ 4
4. 688 ÷ 4
5. 942 ÷ 6
6. 870 ÷ 6
7. 1296 ÷ 8
8. 1176 ÷ 8
9. 1855 ÷ 7
10. 1666 ÷ 7
11. 2376 ÷ 9
12. 3186 ÷ 9
13. 8442 ÷ 6
14. 7818 ÷ 6
15. 7035 ÷ 5
16. 8025 ÷ 5
17. 7532 ÷ 7
18. 7476 ÷ 7
19. 4288 ÷ 8
20. 3584 ÷ 8
21. 34756 ÷ 4
22. 28965 ÷ 5
23. 15894 ÷ 6
24. 34756 ÷ 4
25. 21637 ÷ 7
26. 48568 ÷ 8
27. 45963 ÷ 9
28. 42654 ÷ 6
29. 26352 ÷ 8
30. 46186 ÷ 7
31. 26352 ÷ 8
32. 46186 ÷ 7
33. 34643 ÷ 7
34. 53824 ÷ 8
35. 53790 ÷ 6
36. 42822 ÷ 9
37. 336623 ÷ 7
38. 234522 ÷ 6
39. 412101 ÷ 9
40. 231111 ÷ 9

Section B

1. 739 ÷ 4
2. 598 ÷ 4
3. 876 ÷ 5
4. 985 ÷ 5
5. 952 ÷ 7
6. 897 ÷ 7
7. 874 ÷ 6
8. 777 ÷ 6
9. 2798 ÷ 8
10. 3676 ÷ 8
11. 5967 ÷ 8
12. 6838 ÷ 8
13. 5899 ÷ 9
14. 6897 ÷ 9
15. 8412 ÷ 6
16. 7834 ÷ 6
17. 7458 ÷ 7
18. 7581 ÷ 7
19. 4724 ÷ 8
20. 3871 ÷ 8
21. 35142 ÷ 4
22. 31373 ÷ 4
23. 46169 ÷ 6
24. 45259 ÷ 7
25. 55609 ÷ 8
26. 62342 ÷ 7
27. 61633 ÷ 7
28. 53891 ÷ 8
29. 42822 ÷ 9
30. 52674 ÷ 9
31. 412515 ÷ 6
32. 332753 ÷ 7
33. 232311 ÷ 8
34. 344766 ÷ 9
35. 605554 ÷ 7
36. 512462 ÷ 8
37. 260412 ÷ 9
38. 341161 ÷ 7
39. 312101 ÷ 7
40. 161620 ÷ 9

Section C

1. 3894 ÷ 11
2. 4983 ÷ 11
3. 6768 ÷ 12
4. 7836 ÷ 12
5. 7448 ÷ 14
6. 4956 ÷ 14
7. 3795 ÷ 15
8. 9480 ÷ 15
9. 8384 ÷ 16
10. 3856 ÷ 16
11. 5848 ÷ 17
12. 5525 ÷ 17
13. 2250 ÷ 18
14. 2412 ÷ 18
15. 6156 ÷ 19
16. 8246 ÷ 19
17. 5166 ÷ 21
18. 7287 ÷ 21
19. 7498 ÷ 23
20. 5451 ÷ 23
21. 90376 ÷ 26
22. 90192 ÷ 24
23. 67581 ÷ 27
24. 87783 ÷ 29
25. 50176 ÷ 32
26. 79577 ÷ 31
27. 51084 ÷ 33
28. 53346 ÷ 34
29. 61128 ÷ 36
30. 98013 ÷ 37
31. 142155 ÷ 39
32. 184870 ÷ 38
33. 245028 ÷ 42
34. 261252 ÷ 41
35. 367349 ÷ 43
36. 323752 ÷ 44
37. 276828 ÷ 46
38. 141893 ÷ 47
39. 204357 ÷ 51
40. 343147 ÷ 49

Division Practice ~ 2

Section D

1. 2947 ÷ 12
2. 1963 ÷ 12
3. 4247 ÷ 13
4. 8354 ÷ 13
5. 8121 ÷ 15
6. 5475 ÷ 15
7. 4858 ÷ 14
8. 9135 ÷ 14
9. 4172 ÷ 17
10. 7231 ÷ 17
11. 7834 ÷ 18
12. 9437 ÷ 18
13. 4451 ÷ 19
14. 2511 ÷ 19
15. 7510 ÷ 22
16. 9314 ÷ 22
17. 8946 ÷ 21
18. 7289 ÷ 21
19. 4971 ÷ 23
20. 9591 ÷ 23
21. 39252 ÷ 24
22. 70863 ÷ 26
23. 69596 ÷ 28
24. 87966 ÷ 27
25. 59115 ÷ 32
26. 85451 ÷ 29
27. 70516 ÷ 34
28. 81645 ÷ 33
29. 53196 ÷ 37
30. 60953 ÷ 36
31. 518641 ÷ 38
32. 715133 ÷ 39
33. 715605 ÷ 42
34. 854301 ÷ 41
35. 688040 ÷ 44
36. 711041 ÷ 43
37. 285731 ÷ 47
38. 369671 ÷ 46
39. 211071 ÷ 49
40. 494891 ÷ 51

Section E

1. 142792 ÷ 52
2. 437091 ÷ 53
3. 267575 ÷ 55
4. 198936 ÷ 54
5. 159432 ÷ 56
6. 396036 ÷ 57
7. 428576 ÷ 59
8. 565268 ÷ 58
9. 460489 ÷ 61
10. 355198 ÷ 62
11. 566592 ÷ 64
12. 172935 ÷ 63
13. 546084 ÷ 66
14. 557245 ÷ 65
15. 138288 ÷ 67
16. 185708 ÷ 68
17. 494373 ÷ 71
18. 297114 ÷ 69
19. 461376 ÷ 72
20. 440336 ÷ 73
21. 673134 ÷ 77
22. 485108 ÷ 76
23. 484146 ÷ 78
24. 492565 ÷ 79
25. 766536 ÷ 82
26. 411804 ÷ 81
27. 253316 ÷ 83
28. 795396 ÷ 84
29. 297818 ÷ 86
30. 644496 ÷ 87
31. 830014 ÷ 89
32. 760056 ÷ 88
33. 401856 ÷ 92
34. 214669 ÷ 91
35. 843231 ÷ 93
36. 301552 ÷ 94
37. 308064 ÷ 96
38. 203312 ÷ 97
39. 519453 ÷ 99
40. 624750 ÷ 98

Section F

1. 279558 ÷ 53
2. 377807 ÷ 52
3. 445675 ÷ 54
4. 157536 ÷ 55
5. 187894 ÷ 57
6. 388783 ÷ 56
7. 170894 ÷ 58
8. 284661 ÷ 59
9. 462038 ÷ 62
10. 582085 ÷ 61
11. 171713 ÷ 63
12. 183845 ÷ 64
13. 372152 ÷ 65
14. 188382 ÷ 66
15. 462623 ÷ 68
16. 471371 ÷ 67
17. 277811 ÷ 69
18. 490191 ÷ 71
19. 199784 ÷ 73
20. 573384 ÷ 72
21. 362022 ÷ 76
22. 491624 ÷ 77
23. 569381 ÷ 79
24. 241585 ÷ 78
25. 733411 ÷ 81
26. 699523 ÷ 82
27. 473836 ÷ 84
28. 473791 ÷ 83
29. 319743 ÷ 87
30. 408165 ÷ 86
31. 848021 ÷ 88
32. 343340 ÷ 89
33. 510383 ÷ 91
34. 100903 ÷ 92
35. 551866 ÷ 94
36. 354033 ÷ 93
37. 105253 ÷ 97
38. 832413 ÷ 96
39. 722151 ÷ 98
40. 351132 ÷ 99

2 × 1 = 2
2 × 2 = 4
2 × 3 = 6
2 × 4 = 8
2 × 5 = 10
2 × 6 = 12
2 × 7 = 14
2 × 8 = 16
2 × 9 = 18

3 × 1 = 3
3 × 2 = 6
3 × 3 = 9
3 × 4 = 12
3 × 5 = 15
3 × 6 = 18
3 × 7 = 21
3 × 8 = 24
3 × 9 = 27

4 × 1 = 4
4 × 2 = 8
4 × 3 = 12
4 × 4 = 16
4 × 5 = 20
4 × 6 = 24
4 × 7 = 28
4 × 8 = 32
4 × 9 = 36

5 × 1 = 5
5 × 2 = 10
5 × 3 = 15
5 × 4 = 20
5 × 5 = 25
5 × 6 = 30
5 × 7 = 35
5 × 8 = 40
5 × 9 = 45

6 × 1 = 6
6 × 2 = 12
6 × 3 = 18
6 × 4 = 24
6 × 5 = 30
6 × 6 = 36
6 × 7 = 42
6 × 8 = 48
6 × 9 = 54

7 × 1 = 7
7 × 2 = 14
7 × 3 = 21
7 × 4 = 28
7 × 5 = 35
7 × 6 = 42
7 × 7 = 49
7 × 8 = 56
7 × 9 = 63

8 × 1 = 8
8 × 2 = 16
8 × 3 = 24
8 × 4 = 32
8 × 5 = 40
8 × 6 = 48
8 × 7 = 56
8 × 8 = 64
8 × 9 = 72

9 × 1 = 9
9 × 2 = 18
9 × 3 = 27
9 × 4 = 36
9 × 5 = 45
9 × 6 = 54
9 × 7 = 63
9 × 8 = 72
9 × 9 = 81

Multiplication tables ~ 2

14 × 1 = 14	**23** × 1 = 23	**34** × 1 = 34
14 × 2 = 28	**23** × 2 = 46	**34** × 2 = 68
14 × 3 = 42	**23** × 3 = 69	**34** × 3 = 102
14 × 4 = 56	**23** × 4 = 92	**34** × 4 = 136
14 × 5 = 70	**23** × 5 = 115	**34** × 5 = 170
14 × 6 = 84	**23** × 6 = 138	**34** × 6 = 204
14 × 7 = 98	**23** × 7 = 161	**34** × 7 = 238
14 × 8 = 112	**23** × 8 = 184	**34** × 8 = 272
14 × 9 = 126	**23** × 9 = 207	**34** × 9 = 306
37 × 1 = 37	**44** × 1 = 44	**53** × 1 = 53
37 × 2 = 74	**44** × 2 = 88	**53** × 2 = 106
37 × 3 = 111	**44** × 3 = 132	**53** × 3 = 159
37 × 4 = 148	**44** × 4 = 176	**53** × 4 = 212
37 × 5 = 185	**44** × 5 = 220	**53** × 5 = 265
37 × 6 = 222	**44** × 6 = 264	**53** × 6 = 318
37 × 7 = 259	**44** × 7 = 308	**53** × 7 = 371
37 × 8 = 296	**44** × 8 = 352	**53** × 8 = 424
37 × 9 = 333	**44** × 9 = 396	**53** × 9 = 477
68 × 1 = 68	**87** × 1 = 87	**96** × 1 = 96
68 × 2 = 136	**87** × 2 = 174	**96** × 2 = 192
68 × 3 = 204	**87** × 3 = 261	**96** × 3 = 288
68 × 4 = 272	**87** × 4 = 348	**96** × 4 = 384
68 × 5 = 340	**87** × 5 = 435	**96** × 5 = 480
68 × 6 = 408	**87** × 6 = 522	**96** × 6 = 576
68 × 7 = 476	**87** × 7 = 609	**96** × 7 = 672
68 × 8 = 544	**87** × 8 = 696	**96** × 8 = 768
68 × 9 = 612	**87** × 9 = 783	**96** × 9 = 864

4 × 1 = 4	5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8	9 × 1 = 9
4 × 2 = 8	5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16	9 × 2 = 18
4 × 3 = 12	5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24	9 × 3 = 27
4 × 4 = 16	5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32	9 × 4 = 36
4 × 5 = 20	5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40	9 × 5 = 45
4 × 6 = 24	5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48	9 × 6 = 54
4 × 7 = 28	5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56	9 × 7 = 63
4 × 8 = 32	5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64	9 × 8 = 72
4 × 9 = 36	5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72	9 × 9 = 81

10 × 1 = 10	11 × 1 = 11	12 × 1 = 12	13 × 1 = 13	14 × 1 = 14	15 × 1 = 15
10 × 2 = 20	11 × 2 = 22	12 × 2 = 24	13 × 2 = 26	14 × 2 = 28	15 × 2 = 30
10 × 3 = 30	11 × 3 = 33	12 × 3 = 36	13 × 3 = 39	14 × 3 = 42	15 × 3 = 45
10 × 4 = 40	11 × 4 = 44	12 × 4 = 48	13 × 4 = 52	14 × 4 = 56	15 × 4 = 60
10 × 5 = 50	11 × 5 = 55	12 × 5 = 60	13 × 5 = 65	14 × 5 = 70	15 × 5 = 75
10 × 6 = 60	11 × 6 = 66	12 × 6 = 72	13 × 6 = 78	14 × 6 = 84	15 × 6 = 90
10 × 7 = 70	11 × 7 = 77	12 × 7 = 84	13 × 7 = 91	14 × 7 = 98	15 × 7 = 105
10 × 8 = 80	11 × 8 = 88	12 × 8 = 96	13 × 8 = 104	14 × 8 = 112	15 × 8 = 120
10 × 9 = 90	11 × 9 = 99	12 × 9 = 108	13 × 9 = 117	14 × 9 = 126	15 × 9 = 135

16 × 1 = 16	17 × 1 = 17	18 × 1 = 18	19 × 1 = 19	20 × 1 = 20	21 × 1 = 21
16 × 2 = 32	17 × 2 = 34	18 × 2 = 36	19 × 2 = 38	20 × 2 = 40	21 × 2 = 42
16 × 3 = 48	17 × 3 = 51	18 × 3 = 54	19 × 3 = 57	20 × 3 = 60	21 × 3 = 63
16 × 4 = 64	17 × 4 = 68	18 × 4 = 72	19 × 4 = 76	20 × 4 = 80	21 × 4 = 84
16 × 5 = 80	17 × 5 = 85	18 × 5 = 90	19 × 5 = 95	20 × 5 = 100	21 × 5 = 105
16 × 6 = 96	17 × 6 = 102	18 × 6 = 108	19 × 6 = 114	20 × 6 = 120	21 × 6 = 126
16 × 7 = 112	17 × 7 = 119	18 × 7 = 126	19 × 7 = 133	20 × 7 = 140	21 × 7 = 147
16 × 8 = 128	17 × 8 = 136	18 × 8 = 144	19 × 8 = 152	20 × 8 = 160	21 × 8 = 168
16 × 9 = 144	17 × 9 = 153	18 × 9 = 162	19 × 9 = 171	20 × 9 = 180	21 × 9 = 189

22 × 1 = 22	23 × 1 = 23	24 × 1 = 24	25 × 1 = 25	26 × 1 = 26	27 × 1 = 27
22 × 2 = 44	23 × 2 = 46	24 × 2 = 48	25 × 2 = 50	26 × 2 = 52	27 × 2 = 54
22 × 3 = 66	23 × 3 = 69	24 × 3 = 72	25 × 3 = 75	26 × 3 = 78	27 × 3 = 81
22 × 4 = 88	23 × 4 = 92	24 × 4 = 96	25 × 4 = 100	26 × 4 = 104	27 × 4 = 108
22 × 5 = 110	23 × 5 = 115	24 × 5 = 120	25 × 5 = 125	26 × 5 = 130	27 × 5 = 135
22 × 6 = 132	23 × 6 = 138	24 × 6 = 144	25 × 6 = 150	26 × 6 = 156	27 × 6 = 162
22 × 7 = 154	23 × 7 = 161	24 × 7 = 168	25 × 7 = 175	26 × 7 = 182	27 × 7 = 189
22 × 8 = 176	23 × 8 = 184	24 × 8 = 192	25 × 8 = 200	26 × 8 = 208	27 × 8 = 216
22 × 9 = 198	23 × 9 = 207	24 × 9 = 216	25 × 9 = 225	26 × 9 = 234	27 × 9 = 243

28 × 1 = 28	29 × 1 = 29	30 × 1 = 30	31 × 1 = 31	32 × 1 = 32	33 × 1 = 33
28 × 2 = 56	29 × 2 = 58	30 × 2 = 60	31 × 2 = 62	32 × 2 = 64	33 × 2 = 66
28 × 3 = 84	29 × 3 = 87	30 × 3 = 90	31 × 3 = 93	32 × 3 = 96	33 × 3 = 99
28 × 4 = 112	29 × 4 = 116	30 × 4 = 120	31 × 4 = 124	32 × 4 = 128	33 × 4 = 132
28 × 5 = 140	29 × 5 = 145	30 × 5 = 150	31 × 5 = 155	32 × 5 = 160	33 × 5 = 165
28 × 6 = 168	29 × 6 = 174	30 × 6 = 180	31 × 6 = 186	32 × 6 = 192	33 × 6 = 198
28 × 7 = 196	29 × 7 = 203	30 × 7 = 210	31 × 7 = 217	32 × 7 = 224	33 × 7 = 231
28 × 8 = 224	29 × 8 = 232	30 × 8 = 240	31 × 8 = 248	32 × 8 = 256	33 × 8 = 264
28 × 9 = 252	29 × 9 = 261	30 × 9 = 270	31 × 9 = 279	32 × 9 = 288	33 × 9 = 297

34 × 1 = 34	35 × 1 = 35	36 × 1 = 36	37 × 1 = 37	38 × 1 = 38	39 × 1 = 39
34 × 2 = 68	35 × 2 = 70	36 × 2 = 72	37 × 2 = 74	38 × 2 = 76	39 × 2 = 78
34 × 3 = 102	35 × 3 = 105	36 × 3 = 108	37 × 3 = 111	38 × 3 = 114	39 × 3 = 117
34 × 4 = 136	35 × 4 = 140	36 × 4 = 144	37 × 4 = 148	38 × 4 = 152	39 × 4 = 156
34 × 5 = 170	35 × 5 = 175	36 × 5 = 180	37 × 5 = 185	38 × 5 = 190	39 × 5 = 195
34 × 6 = 204	35 × 6 = 210	36 × 6 = 216	37 × 6 = 222	38 × 6 = 228	39 × 6 = 234
34 × 7 = 238	35 × 7 = 245	36 × 7 = 252	37 × 7 = 259	38 × 7 = 266	39 × 7 = 273
34 × 8 = 272	35 × 8 = 280	36 × 8 = 288	37 × 8 = 296	38 × 8 = 304	39 × 8 = 312
34 × 9 = 306	35 × 9 = 315	36 × 9 = 324	37 × 9 = 333	38 × 9 = 342	39 × 9 = 351

40 × 1 = 40	41 × 1 = 41	42 × 1 = 42	43 × 1 = 43	44 × 1 = 44	45 × 1 = 45
40 × 2 = 80	41 × 2 = 82	42 × 2 = 84	43 × 2 = 86	44 × 2 = 88	45 × 2 = 90
40 × 3 = 120	41 × 3 = 123	42 × 3 = 126	43 × 3 = 129	44 × 3 = 132	45 × 3 = 135
40 × 4 = 160	41 × 4 = 164	42 × 4 = 168	43 × 4 = 172	44 × 4 = 176	45 × 4 = 180
40 × 5 = 200	41 × 5 = 205	42 × 5 = 210	43 × 5 = 215	44 × 5 = 220	45 × 5 = 225
40 × 6 = 240	41 × 6 = 246	42 × 6 = 252	43 × 6 = 258	44 × 6 = 264	45 × 6 = 270
40 × 7 = 280	41 × 7 = 287	42 × 7 = 294	43 × 7 = 301	44 × 7 = 308	45 × 7 = 315
40 × 8 = 320	41 × 8 = 328	42 × 8 = 336	43 × 8 = 344	44 × 8 = 352	45 × 8 = 360
40 × 9 = 360	41 × 9 = 369	42 × 9 = 378	43 × 9 = 387	44 × 9 = 396	45 × 9 = 405

46 × 1 = 46	47 × 1 = 47	48 × 1 = 48	49 × 1 = 49	50 × 1 = 50	51 × 1 = 51
46 × 2 = 92	47 × 2 = 94	48 × 2 = 96	49 × 2 = 98	50 × 2 = 100	51 × 2 = 102
46 × 3 = 138	47 × 3 = 141	48 × 3 = 144	49 × 3 = 147	50 × 3 = 150	51 × 3 = 153
46 × 4 = 184	47 × 4 = 188	48 × 4 = 192	49 × 4 = 196	50 × 4 = 200	51 × 4 = 204
46 × 5 = 230	47 × 5 = 235	48 × 5 = 240	49 × 5 = 245	50 × 5 = 250	51 × 5 = 255
46 × 6 = 276	47 × 6 = 282	48 × 6 = 288	49 × 6 = 294	50 × 6 = 300	51 × 6 = 306
46 × 7 = 322	47 × 7 = 329	48 × 7 = 336	49 × 7 = 343	50 × 7 = 350	51 × 7 = 357
46 × 8 = 368	47 × 8 = 376	48 × 8 = 384	49 × 8 = 392	50 × 8 = 400	51 × 8 = 408
46 × 9 = 414	47 × 9 = 423	48 × 9 = 432	49 × 9 = 441	50 × 9 = 450	51 × 9 = 459

Multiplication tables 52 - 99

52 × 1 = 52	53 × 1 = 53	54 × 1 = 54	55 × 1 = 55	56 × 1 = 56	57 × 1 = 57
52 × 2 = 104	53 × 2 = 106	54 × 2 = 108	55 × 2 = 110	56 × 2 = 112	57 × 2 = 114
52 × 3 = 156	53 × 3 = 159	54 × 3 = 162	55 × 3 = 165	56 × 3 = 168	57 × 3 = 171
52 × 4 = 208	53 × 4 = 212	54 × 4 = 216	55 × 4 = 220	56 × 4 = 224	57 × 4 = 228
52 × 5 = 260	53 × 5 = 265	54 × 5 = 270	55 × 5 = 275	56 × 5 = 280	57 × 5 = 285
52 × 6 = 312	53 × 6 = 318	54 × 6 = 324	55 × 6 = 330	56 × 6 = 336	57 × 6 = 342
52 × 7 = 364	53 × 7 = 371	54 × 7 = 378	55 × 7 = 385	56 × 7 = 392	57 × 7 = 399
52 × 8 = 416	53 × 8 = 424	54 × 8 = 432	55 × 8 = 440	56 × 8 = 448	57 × 8 = 456
52 × 9 = 468	53 × 9 = 477	54 × 9 = 486	55 × 9 = 495	56 × 9 = 504	57 × 9 = 513

58 × 1 = 58	59 × 1 = 59	60 × 1 = 60	61 × 1 = 61	62 × 1 = 62	63 × 1 = 63
58 × 2 = 116	59 × 2 = 118	60 × 2 = 120	61 × 2 = 122	62 × 2 = 124	63 × 2 = 126
58 × 3 = 174	59 × 3 = 177	60 × 3 = 180	61 × 3 = 183	62 × 3 = 186	63 × 3 = 189
58 × 4 = 232	59 × 4 = 236	60 × 4 = 240	61 × 4 = 244	62 × 4 = 248	63 × 4 = 252
58 × 5 = 290	59 × 5 = 295	60 × 5 = 300	61 × 5 = 305	62 × 5 = 310	63 × 5 = 315
58 × 6 = 348	59 × 6 = 354	60 × 6 = 360	61 × 6 = 366	62 × 6 = 372	63 × 6 = 378
58 × 7 = 406	59 × 7 = 413	60 × 7 = 420	61 × 7 = 427	62 × 7 = 434	63 × 7 = 441
58 × 8 = 464	59 × 8 = 472	60 × 8 = 480	61 × 8 = 488	62 × 8 = 496	63 × 8 = 504
58 × 9 = 522	59 × 9 = 531	60 × 9 = 540	61 × 9 = 549	62 × 9 = 558	63 × 9 = 567

64 × 1 = 64	65 × 1 = 65	66 × 1 = 66	67 × 1 = 67	68 × 1 = 68	69 × 1 = 69
64 × 2 = 128	65 × 2 = 130	66 × 2 = 132	67 × 2 = 134	68 × 2 = 136	69 × 2 = 138
64 × 3 = 192	65 × 3 = 195	66 × 3 = 198	67 × 3 = 201	68 × 3 = 204	69 × 3 = 207
64 × 4 = 256	65 × 4 = 260	66 × 4 = 264	67 × 4 = 268	68 × 4 = 272	69 × 4 = 276
64 × 5 = 320	65 × 5 = 325	66 × 5 = 330	67 × 5 = 335	68 × 5 = 340	69 × 5 = 345
64 × 6 = 384	65 × 6 = 390	66 × 6 = 396	67 × 6 = 402	68 × 6 = 408	69 × 6 = 414
64 × 7 = 448	65 × 7 = 455	66 × 7 = 462	67 × 7 = 469	68 × 7 = 476	69 × 7 = 483
64 × 8 = 512	65 × 8 = 520	66 × 8 = 528	67 × 8 = 536	68 × 8 = 544	69 × 8 = 552
64 × 9 = 576	65 × 9 = 585	66 × 9 = 594	67 × 9 = 603	68 × 9 = 612	69 × 9 = 621

70 × 1 = 70	71 × 1 = 71	72 × 1 = 72	73 × 1 = 73	74 × 1 = 74	75 × 1 = 75
70 × 2 = 140	71 × 2 = 142	72 × 2 = 144	73 × 2 = 146	74 × 2 = 148	75 × 2 = 150
70 × 3 = 210	71 × 3 = 213	72 × 3 = 216	73 × 3 = 219	74 × 3 = 222	75 × 3 = 225
70 × 4 = 280	71 × 4 = 284	72 × 4 = 288	73 × 4 = 292	74 × 4 = 296	75 × 4 = 300
70 × 5 = 350	71 × 5 = 355	72 × 5 = 360	73 × 5 = 365	74 × 5 = 370	75 × 5 = 375
70 × 6 = 420	71 × 6 = 426	72 × 6 = 432	73 × 6 = 438	74 × 6 = 444	75 × 6 = 450
70 × 7 = 490	71 × 7 = 497	72 × 7 = 504	73 × 7 = 511	74 × 7 = 518	75 × 7 = 525
70 × 8 = 560	71 × 8 = 568	72 × 8 = 576	73 × 8 = 584	74 × 8 = 592	75 × 8 = 600
70 × 9 = 630	71 × 9 = 639	72 × 9 = 648	73 × 9 = 657	74 × 9 = 666	75 × 9 = 675

76 × 1 = 76	77 × 1 = 77	78 × 1 = 78	79 × 1 = 79	80 × 1 = 80	81 × 1 = 81
76 × 2 = 152	77 × 2 = 154	78 × 2 = 156	79 × 2 = 158	80 × 2 = 160	81 × 2 = 162
76 × 3 = 228	77 × 3 = 231	78 × 3 = 234	79 × 3 = 237	80 × 3 = 240	81 × 3 = 243
76 × 4 = 304	77 × 4 = 308	78 × 4 = 312	79 × 4 = 316	80 × 4 = 320	81 × 4 = 324
76 × 5 = 380	77 × 5 = 385	78 × 5 = 390	79 × 5 = 395	80 × 5 = 400	81 × 5 = 405
76 × 6 = 456	77 × 6 = 462	78 × 6 = 468	79 × 6 = 474	80 × 6 = 480	81 × 6 = 486
76 × 7 = 532	77 × 7 = 539	78 × 7 = 546	79 × 7 = 553	80 × 7 = 560	81 × 7 = 567
76 × 8 = 608	77 × 8 = 616	78 × 8 = 624	79 × 8 = 632	80 × 8 = 640	81 × 8 = 648
76 × 9 = 684	77 × 9 = 693	78 × 9 = 702	79 × 9 = 711	80 × 9 = 720	81 × 9 = 729

82 × 1 = 82	83 × 1 = 83	84 × 1 = 84	85 × 1 = 85	86 × 1 = 86	87 × 1 = 87
82 × 2 = 164	83 × 2 = 166	84 × 2 = 168	85 × 2 = 170	86 × 2 = 172	87 × 2 = 174
82 × 3 = 246	83 × 3 = 249	84 × 3 = 252	85 × 3 = 255	86 × 3 = 258	87 × 3 = 261
82 × 4 = 328	83 × 4 = 332	84 × 4 = 336	85 × 4 = 340	86 × 4 = 344	87 × 4 = 348
82 × 5 = 410	83 × 5 = 415	84 × 5 = 420	85 × 5 = 425	86 × 5 = 430	87 × 5 = 435
82 × 6 = 492	83 × 6 = 498	84 × 6 = 504	85 × 6 = 510	86 × 6 = 516	87 × 6 = 522
82 × 7 = 574	83 × 7 = 581	84 × 7 = 588	85 × 7 = 595	86 × 7 = 602	87 × 7 = 609
82 × 8 = 656	83 × 8 = 664	84 × 8 = 672	85 × 8 = 680	86 × 8 = 688	87 × 8 = 696
82 × 9 = 738	83 × 9 = 747	84 × 9 = 756	85 × 9 = 765	86 × 9 = 774	87 × 9 = 783

88 × 1 = 88	89 × 1 = 89	90 × 1 = 90	91 × 1 = 91	92 × 1 = 92	93 × 1 = 93
88 × 2 = 176	89 × 2 = 178	90 × 2 = 180	91 × 2 = 182	92 × 2 = 184	93 × 2 = 186
88 × 3 = 264	89 × 3 = 267	90 × 3 = 270	91 × 3 = 273	92 × 3 = 276	93 × 3 = 279
88 × 4 = 352	89 × 4 = 356	90 × 4 = 360	91 × 4 = 364	92 × 4 = 368	93 × 4 = 372
88 × 5 = 440	89 × 5 = 445	90 × 5 = 450	91 × 5 = 455	92 × 5 = 460	93 × 5 = 465
88 × 6 = 528	89 × 6 = 534	90 × 6 = 540	91 × 6 = 546	92 × 6 = 552	93 × 6 = 558
88 × 7 = 616	89 × 7 = 623	90 × 7 = 630	91 × 7 = 637	92 × 7 = 644	93 × 7 = 651
88 × 8 = 704	89 × 8 = 712	90 × 8 = 720	91 × 8 = 728	92 × 8 = 736	93 × 8 = 744
88 × 9 = 792	89 × 9 = 801	90 × 9 = 810	91 × 9 = 819	92 × 9 = 828	93 × 9 = 837

94 × 1 = 94	95 × 1 = 95	96 × 1 = 96	97 × 1 = 97	98 × 1 = 98	99 × 1 = 99
94 × 2 = 188	95 × 2 = 190	96 × 2 = 192	97 × 2 = 194	98 × 2 = 196	99 × 2 = 198
94 × 3 = 282	95 × 3 = 285	96 × 3 = 288	97 × 3 = 291	98 × 3 = 294	99 × 3 = 297
94 × 4 = 376	95 × 4 = 380	96 × 4 = 384	97 × 4 = 388	98 × 4 = 392	99 × 4 = 396
94 × 5 = 470	95 × 5 = 475	96 × 5 = 480	97 × 5 = 485	98 × 5 = 490	99 × 5 = 495
94 × 6 = 564	95 × 6 = 570	96 × 6 = 576	97 × 6 = 582	98 × 6 = 588	99 × 6 = 594
94 × 7 = 658	95 × 7 = 665	96 × 7 = 672	97 × 7 = 679	98 × 7 = 686	99 × 7 = 693
94 × 8 = 752	95 × 8 = 760	96 × 8 = 768	97 × 8 = 776	98 × 8 = 784	99 × 8 = 792
94 × 9 = 846	95 × 9 = 855	96 × 9 = 864	97 × 9 = 873	98 × 9 = 882	99 × 9 = 891

A Parliamentary Division

In 1866, in the House of Commons in the United Kingdom, a Reform Bill was being debated which was aimed at giving the vote to many hundreds of thousands of men[1]. One of the arguments concerned whether or not the vote should only be given to those who could pass some sort of educational test. The then *Chancellor of the Exchequer* (William Gladstone) was against this idea and, during the course of his speech, said

> "Putting aside subtraction and multiplication, I should like to know how many of the labouring classes can pass an examination in division of money, or how many members of this house can pass such an examination. If I give the sum of £1,330 17*s* 6*d* and tell members of this House to divide it by £2 13*s* 8*d* I want to know how many would do it."
>
> *Mr Hunt:* Six hundred and fifty eight.[2]
>
> *The Chancellor of the Exchequer:* There are not three or four in this House who could do it. I would day there are not thirty or forty, without the least fear of contradiction. I will go further and say it is not necessary that they should; and that they may be admirable members of this House without being able to work such a sum.
>
> *Lord Robert Montagu:* You cannot divide by £2 13*s* 8*d*. [Laughter]
>
> *The Chancellor of the Exchequer:* One illustration is better than a thousand arguments. The noble Lord is one of the more promising financial members of the House and he tells us positively that division of money is a thing that cannot be done.

Later, Lord Montagu offered this explanation of what he had really meant:

> "With regard to the sum of division which the Right Honourable Gentleman has suggested, it was quite possible to divide the sum of money, but not by *money*. How could one divide money by £2 13*s* 8*d*? The question might be asked, 'How many times 2 *shillings* will go into £1?' but that was not dividing by money; it was simply dividing 20 by 2. He might be asked, 'How many times will 6*s* 8*d* go into a pound?', but that was merely dividing 240 by 80."

[1] This Bill was eventually passed in 1867 giving the vote to about another one-million men, but still excluding many. Reform Bills of later years gradually increased the number of men eligible to vote. Women (after a bitter struggle lasting over 50 years) were finally given the vote in 1918, and then only those over 30 years of age and falling within certain categories. They did not achieve parity with men until 1928. This may be contrasted with New Zealand where women had had the right to vote from 1893, and Australia since 1901.

[2] This was the number of MP's in the House of Commons

Author's reminiscence

My primary/secondary schooling took place during the period 1935 to 1945, when the use of calculators of any sort was unknown in schools, and all arithmetic was done 'by hand', (logarithms in the later stages). One of my strongest memories of those days is of doing massive sums such as

3 *tons* 14 *cwt* 5 *stones* 9 *pounds* ÷ 3 *cwt* 2 *stones* 4 *pounds* 5 *ounces*

and there were similar things for length, area, weight, capacity, as well as money, just like Gladstone's example above. There were also an awful lot of them! I quite liked them, but then I was good at arithmetic, I do not recall my enthusiasm being shared by many of my fellow pupils.

Of course we also did sums where the divisor was a number, but they are a little easier because units are dealt with as they arise and you do not have reduce both to a common unit before starting work on the division. So, I appreciated that partition was 'better' than quotition even if I did not know what they were called.

As usual this had little to do with what happened in the real world. There, the people who had reason to actually need such things used 'Ready Reckoners' of which there hundreds to choose from for all sorts of purposes.