

Introduction

The principal purpose of this unit is to provide several ideas which those engaged in teaching mathematics could use with their pupils, using a reasonably familiar artefact - a set of dominoes. It is not a unit which is intended to be worked through from 1 to the end. To get the best from this material demands some familiarity with it so that choices can be made, and a structure imposed, which suits the environment in which it is to be used. The material has been used in a variety of scenarios:

- As 'one-offs' to fill in some odd-moments.
- As an 'intense' piece of work continued over a few weeks to generate a major folder of work.
- As a 'background' project to be done at intervals during a term as other work allows.
- As a 'continuous' project to be worked at for a set time each week, supplemented by a certain amount of homework time.
- As a 'summer-school' activity to motivate some arithmetic.
- As a 'bridging' project between primary and secondary schools to provide some degree of continuity. Pupils started the work during the last few weeks of their primary stage, and then picked up their folders and resumed work when they joined the secondary school. This requires a lot of collaborative work in the preparation, but was generally thought to be of benefit to all (teachers and pupils).

The underlying purpose for doing most of this work, of course, is to provide plenty of practice in the allimportant handling of small numbers in an attempt to make that level of arithmetic almost instinctive. But there are other important skills in the work, principally those of logical reasoning and spatial awareness.

Getting Started

Clearly a first requirement, if this is to be done by all, is that everyone has her or his own set of dominoes. Master sets are provided here, of two different sizes, which can be copied on card and then cut up. Some particular points to note are:

- One of the larger sets, printed on acetate, is good for use on an ohp.
- The smaller sets are fitted two to a sheet and the pair of blank dominoes in the middle are intended as spares, to be kept back and only filled-in when a replacement is needed for some reason.
- Sets clearly need to be kept complete and not mixed up. To that end, have as many different colours as possible for the card used in the printing, and make pupils identify their own sets as they make them with suitable marks on the back.
- The set with circles instead of dots are liked by many as they can colour-in the circles (match colours to numbers?). Might be better printed on white card.
- Some pupils like to bring their own 'proper' sets of dominoes. Remember the downside to that is the hassle that can accompany the business of having personal property in the classroom (losing one of a personal set can waste a lot of time). Also it raises the noise-level appreciably. Your call.
- Rubber-bands are useful for holding sets together and an (old) envelope with a name on it in which the bundle is placed.

Whilst copying the materials needed, including ohp transparencies, it might be a good idea to stock-up on Domino Grid Paper.

Nowadays we cannot assume that everyone is familiar with dominoes, as we might once have done. not that there is much to grasp, but it does need to be made clear that each domino is divided into two equal parts (usually squares) and that each those parts shows a number by means of a set of dots (or is blank). The numbers range from 0 to 6 in this case. And there are 28 dominoes in the complete set, no two of which are alike.

In traditional usage the dominoes are often referred as **stones**, and the marks on them as **spots**, **dots** or **pips**.

It is important to get the notation sorted out, especially that used throughout the work given here. In the listing and recording of dominoes, only the two numbers are used, separted by a hyphen or dash, and the smallest number is always given first (on the left).

So, 0 - 3 2 - 3 4 - 6 1 - 4 5 - 5 0 - 1 1 - 2 shows seven dominoes.

When making a list it is ordered on the first number and then, if needed, on the second number. So the above set of seven dominoes would be listed as

0-1 0-3 1-2 1-4 2-3 4-6 5-5

In this way a repetition is quickly identified. The ohp set of dominoes marked with dots, in conjunction with those marked with numbers, might be useful here.

A detail which must be cleared up at the outset is that in most of this work (but not all) we ignore the rule, usually employed in games played with dominoes, that where dominoes touch the numbers must match.

It has happened (but only in a few cases) that work set as homework has been ruled as being impossible (by a parent) since matching could clearly not be done with the set listed.

An Alternative Starting Point

Rather than just cutting the dominoes out and then working with them, it is worthwhile considering an investigative approach, if it is suitable for the group concerned.

Define a domino as two squares fastened together by a common edge to form an oblong, with a number in each square. The numbers may be the same or different. A set of dominoes is made by first deciding on a number n and then generating all possible pairs (order is immaterial) from 0 to n.

So,	n = 0 generates only	0 - 0				A set of 1
	n = 1 generates	0 - 0	0 - 1	1 - 1		A set of 3
	n = 2 generates	0 - 0	0 - 1		2 - 2	and so on
Each	set is identified as being	g the dou	ble - <i>n</i> s	et.		

This then could lead to things like:

- Write out the different sets, up to some given value of *n*.
- How many dominoes are there in each set?
- How many doubles are there in each set?
- What is the total number of dots in each set?
- Can formulas be found for any of the above, based on the value of *n*?
- The traditional set is the double-six set (which has ? dominoes) but double-nine sets are also available commercially. How many dominoes are there in that set?

A Homework Start

One way of avoiding the need for having a large number of scissors all at once, the waste of a lot of time, and the subsequent clearing-up afterwards, is to get the job done at home. Each pupil is given a card with a set of dominoes printed on it and some clear instructions on the lines of:

- cut carefully
- identify each one on the back
- tie them in a neat bundle with a piece of cotton (or elastic band)
- put them in an old envelope.

In that case it is a good idea to accompany it with a sheet of questions or activities to help pupils gain some familiarity with them before approaching the work to be done in the classroom. (And don't forget to plan for those who fail to bring them back on the due date!) The work could be presented as a form to be filled in with responses to questions like:

- How many dominoes are there in the double-six set?
- How many doubles are there?
- How many 1's or 2's or 3's etc are there?
- How many dominoes have a **total** of 10 dots on them?
- How many dominoes have a **total** of 5 dots on them?
- How many have an **odd total** on them?
- How many with a difference between the two halves of 3?
- How many dots altogether in the complete set?
- What is the greatest/least number of dots to be found on 3 dominoes?

... and so on

Miniaturization

All of the work given here is based on the premise that the full double-six set of dominoes is available. In some cases (say with younger pupils) this can be a little daunting, and the doing of the task can be made much more difficult by the need to work with, and sort through, so many pieces. This is pity, but can be overcome by preparing some work based on smaller sets. After a sudy of the things done in this unit, it is not hard (but does take a little time) to produce similar problems which use smaller sets. Even so few as the six dominoes of the double-two set can be used. In fact, it is not hard to envisage a set of worksheets ranging from the double-two to the double-five set, but that has not been done here.

Marking

This can be overwhelming in both size and complexity if not thought about at the beginning. It is very easy, enjoyable even, to run this as a project over several lessons with very little teacher-input being needed. But the price to be paid at the end in marking-time is horrendous. There are so many alternative answers and, even if the arithmetic is correct, how can you ever manage to check that no domino has been used twice in fulfilling cetain requirements?

There is no single way of dealing with this, but a strategy should be considered before embarking on the work. A process of continuous monitoring (and ticking) of work in progress in the classroom is one possibility. Also, with larger configurations, it can be a big help to rule that, when it is made with the dominoes it must be seen and checked by you (and recorded). Get organized. Be prepared!

Comments on the Worksheets

Most of the sheets are self-explanatory. However, in a few cases, terminology is used that has only been fully explained on a previous sheet, so it will be necessary to look for this when selecting the work and, if necessary, deliver some supplementary instructions. The sheet **Domino Investigations** in particular assumes quite a bit of previous knowledge.

The purpose of supplying a figure (or two) in the answer is to help of course. However, it does have a bonus in that it can sometimes make sure that the answer is unique (if there would have been alternatives without it) and that assists the marking.

* Hollow Domino Squares. Concerned only with addition. There are two sheets on these plus more questions on the Domino Investigations sheet. Further questions that could be used at some time are

For any given size of square (or rectangle) and specified side-total, what is either the greatest, or the least, total number of dots required to achieve it? Some reflection will soon reveal how attacks on this should be organized. Establish class records for these? What is the 'largest' Hollow Domino Square you can make?

Domino Additions Still dealing with relatively easy addition. Difficulties can arise when 'carry figures' are needed. Problems like these are easy to make up, and could be stated without the answer grid, just make sure that they can be made with dominoes.

Similar things can be done with subtraction and multiplication sums.

* **Magic Domino Squares** Still addition, but much more demanding with so many totals to be found at once. If a major challenge is required try this

It is possible to arrange the complete set to make a Magic Square, but only with a little 'cheat'. Note first that 28 dominoes cannot be packed together to make a square. However, if it is done in such a way that one complete outer row or column is composed of blanks, then the rest of the dominoes can be seen to form a 7 by 7 array of numbers. And this square array can be made into a Magic Square in many different ways. All of them will have a magic total of 24. Find one.

* **Domino Multiplication Squares** Good for practising tables. As an extension -*How many different multiplication squares (using 4 dominoes) are possible?*

Hollow Domino Rectangles. More addition. All the rectangles given here are oblongs of course and it might be worthwhile pointing out that the Hollow Domino Squares are also a sub-set of the Rectangles. *What is the 'largest' Hollow Domino Oblong you can find?*

Domino Investigations A recap of what has gone before, but there are no pre-drawn answer grids. This is where the Domino Grid Paper would come in useful.

Domino Ray Diagram ~ 1. More addition. But note that the matching rule *is* used as well.

Domino Ray Diagram ~ 2. As for previous, but is a little harder. The total needed for each ray is not given, but it can be worked out. The 7 totals must be the same and no domino is counted in twice. The total of all the dots on the dominoes is 168. So, each ray must total $168 \div 7 = 24$.

And that completes the arithmetical portion of the material given here.

* indicates the introductory explanation can be done using the ohp, and a suitable master is available.

Domino Configurations The difficulty of these puzzles is almost directly in proportion to the number of dominoes used, though how many different numbers are called for also has some bearing. Extensions are another puzzle (which is NOT easy!) and a game-like activity.

From a full set of dominoes, take out all the 'doubles' and also 0 - 6, 1 - 2 and 1 - 4. Arrange the 18 dominoes left in the shape of a complete square so that the same two numbers are not in line with each other, either in the rows or in the columns.

Two players each make a configuration (of some agreed size, but not too big) and copy it down to prove it can be done. Each one then writes out the numbers, without drawing the dominoes, and they exchange those. They then try to find an arrangement of dominoes that fits - it does not have to be the same as the original, but all the numbers must be in their correct places.

Domino Tilings Here the numbers play no part whatsoever and the dominoes are merely used as tiles to make or cover shapes. Once the requirement has been understood, the puzzles given here are only of moderate difficulty. Some related activities are:

How many different ways can the dominoes be packed together to make an m by n rectangle? The smallest values worth using are 3 by 4. Of course it could be that m = n.

How many different ways can 8 dominoes be arranged to make a square?

Fault-free Packing. The diagram shows 6 dominoes packed together to make a rectangle. It can be seen that the middle line runs from one edge of the rectangle to the opposite edge. This can be thought of as a 'fault-line' since the rectangle could be broken into two parts along a single straight line. In fact it is impossible to make a faultfree rectangle with only 6 dominoes. Find the smallest fault-free rectangle that can be made.



Miscellany

A few other activities or challenges. Each uses all the dominoes of a Double-Six set.

- From the full set, divide them into 7 groups of 4. Make each group into a Hollow Square, which ignore side-totals, but which do 'touch and match'.
- Use the complete set to make a Hollow Domino Square (there are many possibilities).
- Make 4 Hollow Rectangles. Touching dominoes must match and there must be the same **total** number of dots in each rectangle.
- Divide the set into 4 groups of 7. Make a Hollow Domino Rectangle with each group so that all the Rectangles have the **same** side-totals.
- Make two Hollow Domino Squares using 10 dominoes for one and 18 for the other, with both squares having the same side-totals.
- Make four Hollow Domino Squares, using 4, 6, 8 and 10 dominoes respectively, with all the squares having the same side-totals.

And then there is the Double-Nine set!!

Further Activities

Consider other shapes for dominoes, what about triangles, squares, pentagons, hexagons?



Why stop at two squares making a domino? Use three squares to make a 'tromino', and need the squares necessarily be arranged in a straight line?

Do the ~ominoes (whatever their shape) have to have numbers or dots on them? Would other symbols be useful sometimes? What about designing a set for young children?

Games

It would be a great shame to do a lot of work with dominoes and never actually play any games with them. So, make sure there is some opportunity for that.

Get pupils to ask at home, as to what games are known and how they are played. Reporting back with either a written or oral account of the rules for playing a particular game (or one of its versions) is an excellent excercise in communication.

Merely collecting the names of different games can be fun, as some of them can have such intriguing names. Like: Muggins, Five-up, All Threes, Block, Matador, Sniff, Seven-toed Pete, and many others. But do seek some evidence that the game actually exists.

A quick game that is easy to explain is *Take Most* for two players. The dominoes are placed, mixed-up and face-down, in a central pool between them. Each draws one domino and they then turn them over and compare them. Whichever player has the higher (or lower) value according to **the rule** takes both of them. If they are equal in value, players merely keep the one domino drawn. When the pool is empty, the player with the most dominoes is the winner. Simple. Variety (and usefulness) is provided by changing **the rule** for successive games. It could be: "add the two numbers on the domino"; "find the difference"; "multiply them together"; "square the larger and subtract the smaller"; "treat them as a fraction (smaller over larger)" and many other things that an inventive mind can conjure up - always bearing in mind the abilities of the group.

Other Sources of Information

There always several books in print on the topic of dominoes. Mainly, of course, these are concerned with games and their rules.

However, the Web is undoubtedly the 'best' source of all. For example, using the seach-engine **www.google.com**

in a recent search turned up the following numbers of entries for different search parameters

dominoes	82,000
dominoes+games	30,000
"domino games"	1,100
dominoes+activities	12,000
dominoes+puzzles	6,000
"domino puzzles"	60
dominoes+matador	150

Of course they are of very varying degrees of usefulness. There is an awful lot of commercial stuff such as books and sets of dominoes. But there are programs for playing games online, some historical accounts, descriptions of games, and contributions from several school-sites with suggestions for activities as well as examples of work done. A very worthwhile browse for all who are interested. Notice the need to choose the search parameters, as well as the way they are formatted, with some care.

Hollow Domino Squares ~ 1

A Hollow Domino Square is made by arranging some dominoes so that they touch each other, making a square around the outside with a square 'hole' in the middle. The smallest possible is made with just 4 dominoes, and one of these is shown on the right, but they can be much bigger. The problems given below use this shape.



Consider the 4 dominoes 0 - 6, 2 - 2, 3 - 4, 4 - 5These can be arranged to make a Hollow Domino Square like that shown which has the property that the 3 numbers along each of the 4 sides all add up to the same total.

3+4+2=9 2+2+5=9 3+6+0=9 0+4+5=9

3	4	2
6		2
0	4	5

In the following problems, a Hollow Domino Square is given with, underneath each, a list of the 4 dominoes used to make it. Find how the dominoes must be arranged to make each one so that, in every case, the total of the 3 numbers along each side is **10**. Some numbers have been placed already.



Hollow Domino Squares ~ 2

In the following problems, find how the dominoes listed must be arranged to make each Hollow Domino Square so that, in every case, the total of the 3 numbers along each side is **14**. Where no domino is listed, find a suitable domino and write in its numbers on the list.



In the following problems, find how the dominoes listed must be arranged to make each Hollow Domino Square so that, in every case, the total of the numbers along each side is **12**.

Domino Additions

which could be shown using dominoes like this 1 2

$$\frac{3}{4}$$
 $\frac{4}{6}$

+

Here is an addition sum

4

+

$$+ \begin{array}{c} 1 \\ 3 \\ \hline 4 \\ \hline 4 \\ \hline 6 \\ \end{array}$$

That is easy. It is not so easy when given the dominoes to make the addition sum. For example, given 1 - 4, 2 - 3, 4 - 6 they can arranged as

which shows the sum 2|3 2 3

$$\frac{4}{6} \frac{1}{4}$$

<u>4 1</u> + 6 4

Fill in these grids to show how the dominoes listed can be arranged as an addition sum.

2 - 4

4-5 5-5

1 - 5

2 - 3

1 - 6

4 - 6

4 - 5

6 - 6

5 - 6

Magic Domino Squares

A Magic Square is a square array of numbers in which the total of every row, every column, and both the diagonals adds up to the same total. Here we deal with problems of making Magic Squares using dominoes. *Usually in Magic Squares, you are not allowed to use the same number twice, but we cannot have that rule here since there are only seven numbers available (0 to 6) on the dominoes.* Example: given the eight dominoes

0 - 0, 0 - 1, 0 - 2, 0 - 3, 1 - 1, 1 - 2, 1 - 3, 2 - 3and told that the Magic Total is 5 we can make the Magic Domino Square shown on the right.

1		3	0		1
3		0	2		0
1		2	1		1
0		0	2		3

Complete the following grids to show how the dominoes listed can be arranged to make Magic Domino Squares.

Domino Multiplication Squares

Consider the 4 dominoes 1 - 2, 1 - 6, 2 - 2, 2 - 3, These can be arranged to make a square with a hole in the middle like that shown on the right, which has the property that the **three** numbers along each of the four sides when multiplied together all give the same answer.

$$2 \times 1 \times 6 = 12$$

 $6 \times 1 \times 2 = 12$
 $2 \times 2 \times 3 = 12$
 $3 \times 2 \times 2 = 12$

2	1	6
2	×	1
3	2	2

Complete each of the following grids to show how the dominoes listed can be used to produce a Domino Multiplication Square whose four sides all make the same answer.

Hollow Domino Rectangles

In the following problems, find how the dominoes listed must be arranged to make each Hollow Domino Rectangle so that, in every case, the side-totals are those given.

Domino Investigations

Section A

ALL the Hollow Domino Squares referred in this section use 4 dominoes.

1. Find and draw a Hollow Domino Square which

a) has a side-total of 6	e) contains 2 'doubles
b) has the smallest possible side-total	f) has a side-total of 12
c) contains 1 'double'	g) contains 3 'doubles'
d) has a side-total of 8	h) contains 3 'blanks'.

- Find and draw as may different Hollow Domino squares as you can having a side-total of 15.
 One Hollow Domino Square can be said to be different to another, provided it uses at least one domino which is different.
- 3. What is the largest side-total you can find for a Hollow Domino Square?
- 4. From the complete double-six set of dominoes, make as many Hollow Domino Squares as you can, without using any domino twice. *Each complete Hollow Domino Square may have a different side-total to the others you make.*

Section B

ALL the Hollow Domino Squares referred in this section use 8 dominoes.

- 5. Use all the 'doubles' and one other domino of your choice to make a Hollow Domino Square having a side-total of 14.
- 6. Find all the dominoes which have a total of 6 spots. Use those, plus four others of your choice, to make a Hollow Domino Square having a side-total of 10.
- 7. Using only those dominoes which have at least 1 spot on them, but also have less than 5 spots, make a Hollow Domino Square.
- 8. Use all the dominoes having 'blanks', plus one other of your choice, to make a Hollow Domino Square.
- 9. Use the dominoes 0 6, 1 6, 2 2, 2 5, 3 3, 3 4, 3 5, 5 5, to make a Hollow Domino Square having a side-total of 17.
- 10. Use the dominoes 1 6, 2 4, 3 3, 4 4, 4 5, 4 6, 5 5, 6 6, to make a Hollow Domino Square having a side-total of 20.

Section C

12. Make Domino Multiplication Squares from each of these sets

a) 1 - 1,	1 - 3,	1 - 6,	2 - 3
b) 1 - 6,	2 - 3,	3 - 3,	3 - 6
c) 1 - 4,	1 - 6,	2 - 3,	2 - 4

- 13. What is the largest answer you can find (using only 4 dominoes) for a Domino Multiplication Square.
- 14. Use the dominoes 2 2, 2 3, 2 4, 2 5, 3 3, 3 4, 4 4, 4 5 to make a Magic Domino Square.

Domino Ray Diagram ~ 1

Use all 28 dominoes of the double-6 set to make this diagram. Where dominoes touch they must **match** (except at the centre). The total of the dots in each of the 8 rays must be 21. Record your result on this sheet.

Domino Ray Diagram ~ 2

Use all 28 dominoes of the double-6 set to make this diagram. Where dominoes touch they must **match** (except at the centre). The total of the dots in each of the 7 rays must be the same. Record your result on this sheet.

Domino Configurations

Each of the following drawings can be made by using the correct number of dominoes. However, it is not shown just how the dominoes are arranged. Find how the dominoes need to be arranged and draw in the necessary lines to show them. Note that no domino can be used twice in the same drawing.

$$\begin{array}{c|c}
 3. \\
 \hline
 2 & 1 & 0 & 1 \\
 2 & 2 & 2 & 1 \\
 \hline
 2 & 2 & 1
 \end{array}$$

6.				
4	4	1	1	1
4	4	2	2	1
4	2	2	2	1
5	5	3	3	3

7.

8. ()() ()

Domino Tilings

Each of the shapes below is made up of squares as shown by the lines.

Some of the shapes have holes in them, indicated by shading.

Now, ignoring the spots on the dominoes, they can be thought as tiles which are made up of 2 squares side by side, and such 'tiles' could be used to make these shapes. Well they could make some, but not all. Which of these shapes could be made using domino 'tiles'?

Domino Knots

Each of the shapes drawn below uses all 28 dominoes of the double-6 set, and where dominoes touch they **match**. Find out how this can be done and record your result on this sheet.

Hollow Domino Square

Solution

3	4	2
6		2
0	4	5

Magic Domino Square

0 - 0 0 - 1 0 - 2 0 - 3 1 - 1 1 - 2 1 - 3 2 - 3 Magic Total 5

Solution

1	3	0	l	1
3	0	2		0
1	2	1		1
0	0	2		3

Domino Multiplication Square

Solution

2	1	6
2	×	1
3	2	2

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Hollow Domino Squares

Domino Grid Paper

Q	Q	Q	Q	Q	Q	Q
0			3	4	Ŋ	0
0	Ŋ	Ŋ	Ŋ	Ŋ	Ŋ	Ŋ
0	0			3	4	Ŋ
'		L	±	L	L	' <u> </u>
	~	4	4	4	4	4
	-	0 4	7	<u>く</u> 4	6 4	4 4
			() () () () () () () () () () () () () (8 2 4	8 8 4	8 4 4