## The Dartboard

Yet another way of encouraging the practice of some arithmetic.
Though darts and dartboards are a familiar enough idea, not everyone will know exactly how the dartboard 'works'. So, a preliminary talk-through will be needed to ensure there is adequate understanding of the scoring principles before embarking on any of this work. To help in this there is a large drawing of a board from which an ohp transparency can be made to aid in explaining the scoring system. Doubles, trebles, inner bull ( $=50$ or double 25 ) and outer bull ( $=25$ ) will all have to be covered. Some oral work will serve as a check. Revealing or concealing the extensions of the $2 \times$ and $3 \times$ tables at the bottom of the sheet is a matter of choice.

Though a range of activities are given or suggested here only one or two will be suitable for use with any particular group of pupils.

There is a single sheet which provides 6 copies of the dartboard. One use for this could be to colour-in areas in which the score (with a single dart) would be

- an odd number
- a multiple of 3
- a prime number
- a multiple of 2
- a multiple of 5
- a multiple of 7

These same sheets could also be used to play some suitable games. Probably best if they are two-player games. Like players take turns marking off a cell and totalling their own scores as they go. No cell can be used twice and, just as in a real game, the start and finish must be on a double. This should be a game in which the first player always wins. Better then, instead of allowing a completely random choice of cell, after the first cell is occupied the next cell to be occupied must be only one move away from the previous one. In this case 'one move' means that only one line must be crossed, no more - and not an intersection of 2 lines either. Thinking about the finishing double needs to be undertaken in good time. Invite pupils to invent their own games.

Some time could be spent on discussing the design of the board with regard to the arrangement of the numbers. It is clear what the original designer(s) had in mind (to penalise errors) but are the penalties equitable over the entire board, or is there a "better" way?

Ignoring, for the moment, the fact that the 'cells' differ in size; if 1 dart lands in the scoring area, what are the probabilities for any particular score? Group the totals into a suitable class-interval (say 5) and show this on a bar chart. This investigation would, of course, be much better done with areas.

Anyone interested in doing some programming could also find a lot of things to do here.

## Some Relevant Figures

The board is defined by 6 concentric circles and 20 equal-sized annular sectors. The sectors do not run right through to the centre of the board, but stop at outer edge of the circle enclosing the outer bull. The respective diameters, in cm , of the circles are given in The Dartboard $\sim 3$

The rather awkward-looking numbers arise from the fact that they are converted from the original specifications which were, of course, in inches.

There are 82 distinct scoring areas but only 6 different shapes and sizes.
With 3 darts, nearly all the scores from 1 to 180 are possible, but there are 9 scores between 160 and 180 which cannot be obtained. Most of the scores can be obtained in more than one way - 60 is the score which can be obtained in the greatest number of different ways (627) and there are 41,664 different ways altogether of scoring.

Games are usually played to a total of 301,501 and 1001 (for team games).

## The Dartboard

## Some historic and Social Notes

Darts are a very old idea. They were, and in some places still are, used as a short-range weapon alongside the spear. There is no record of how they came to be brought indoors for a game, but it is very easy to conjecture on how it came about. References to their use in actually playing games are very scattered and usually do no more than give a passing mention. For example:

A 12th century note on amusements practised by young Londoners on holiday mentions "the casting of stones, darts and other missive weapons". [But were they playing games?] Later, the court of Charles VI of France (c.1400) is recorded as amusing itself with "wrastling and the dart". It is also known that Anne Boleyn (c.1530) gave Henry VIII a set of darts as a present. [Perhaps she made the mistake of winning their first game?] The Pilgrim Fathers (1620) are said to have taken darts with them to the New World.

Coming to more modern times, the game of darts was certainly widely played in public houses in the early 1900's. However, for the first half of the 20th century there were wide variations in the boards and rules used for playing the game. Not until about 1930 did some degree of standardisation start to taken place. And this came about through the introduction of national competitions, ofter sponsored by newspapers. There are now very well-defined rules covering all aspects of the game, and it has become international in scope.

For further information try doing a search on the Web, there is lots there.

## Some variations of the Dartboard

The modern standard or "London" board has a a overall circular scoring area of 34.2 cm diameter and is shown elsewhere throughout this unit. Other boards that have been used are

The "Manchester" has only a 25 cm diameter scoring circle. Compared with the standard, it has no trebles ring, the numbering order is different, the doubles ring is narrower and the bull is smaller. It is shown in the diagram on the right.

The "Yorkshire" has the same size and numbering as a standard board but with no trebles ring or outer bull.
The "Fives" board



The "Target" board shown on the right probably exists now only in memory. But was likely just what the very earliest boards were like.


The line from which the darts are thrown (about 8 or 9 feet) is a set distance measured obviously from the plane of the dart-board. [Obvious?] There is a recorded instance of one league in which the distance of the throwing-line was measured in a straight line from the bull. [Perhaps the organisers were closet Pythagoreans?] This throwing line is known as the 'oche' (pronounced ockey)

## The Dartboard ~1

Complete the table to show what scores can be obtained from throwing a single dart, and how it may be done. Note that not all of the scores listed can be obtained from one dart. The first four have been filled in.

| Score | Single | Double | Treble |
| :---: | :---: | :---: | :---: |
| 1 | 1 |  |  |
| 2 | 2 | 1 |  |
| 3 | 3 |  | 1 |
| 4 | 4 | 2 |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
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| 26 |  |  |  |
| 27 |  |  |  |
| 28 |  |  |  |
| 29 |  |  |  |
| 30 |  |  |  |


| Score | Single | Double | Treble |
| :---: | :---: | :---: | :---: |
| 31 |  |  |  |
| 32 |  |  |  |
| 33 |  |  |  |
| 34 |  |  |  |
| 35 |  |  |  |
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| 57 |  |  |  |
| 58 |  |  |  |
| 59 |  |  |  |
| 60 |  |  |  |



The Dartboard ~ 2
In all of these questions, assume that every dart thrown lands in a scoring area of the board.
Abbreviations used are $d, t, b, o$ which stand for 'double', 'treble', 'bull' and 'outer' respectively.

So, d3 would be a double 3 (=6)
$t 5$ would be a treble $5(=15)$
$b \quad$ would be a score of $50(=d 25)$
o would be a score of 25
A number by itself is a single score.
So, three darts which land in
$7, d 3, t 5$ means $7+6+15$
for a total score of 28
When looking for 'different' throws, changing the order does not make any difference.

So, $d 3,7, t 5$ is the same as $7, d 3, t 5$
but, $\quad 6,7, t 5$ is different from $d 3,7, t 5$

1. What are the highest and lowest scores possible with
(a) 1 dart
(b) 2 darts
(c) 3 darts?
2. With 1 dart it is possible to score 8 in two different ways - as 8 or as $d 4$ What score is it possible to get in three different ways with 1 dart?
3. With 2 darts it is possible to score 5 in six different ways. These are

$$
\begin{array}{lll}
1+4 & 2+3 & 1+d 2 \\
d 1+3 & d 1+t 1 & 2+t 1
\end{array}
$$

With 2 darts it is possible to score 7 in nine different ways. List them.
4. In how many different ways is it possible to score 6 with 3 darts?
5. Copy and complete this table to show in how many different ways each of the scores from 1 to 10 can be obtained using 1,2 and 3 darts respectively.

| Score | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 dart | 1 | 2 |  |  |  |  |  |  |  |  |
| 2 darts | 0 | 1 | 2 |  |  |  |  |  | 12 |  |
| 3 darts | 0 | 0 | 1 | 2 |  |  |  | 21 |  |  |

6. The rules of darts require that a player must start and finish with a double.

List how a player might achieve the following totals in the stated (minimum) number of throws.
(a) 301 in 6 throws
(b) 501 in 9
(c) 1001 in 7 throws.
7. With 3 darts all scores up 160 are obtainable (except $0,1,2$ if all must score) and the maximum possible is $180(t 20, t 20, t 20)$. However, there are some scores between 160 and 180 which cannot be made. What are they?
8. There are 20 sectors on the board, so any 5 adjacent sectors make up a quarter of the board. Which quarter of the board gives the highest total when the numbers around its outside edge are added up?


## The Dartboard ~ 3

The six circles of a dartboard, from largest to smallest, have the following diameters.
All are given in cm to 2 decimal places.
33.65
31.75
20.96
19.05
3.18
1.27

1. There are 82 distinct scoring areas on the dartboard, but only 6 different shapes and sizes. These 6 are identified on the diagram above by the letters ' $a$ ' to ' $f$ '. Calculate the areas of each of these 6

Calculate the total area in which, with a single dart:
2. Any value at all (from 1 to 60 ) is scored
3. A 'double' is scored.
4. A 'treble' is scored.
5. $\mathbf{6 0}$ is scored
6. $\mathbf{3 8}$ is scored
7. $\mathbf{1}$ is scored
8. $\mathbf{1 7}$ is scored
9. $\mathbf{3}$ is scored
10. $\mathbf{2}$ is scored
11. more than $\mathbf{4 1}$ is scored
12. less than $\mathbf{2 3}$ is scored
13. When a single dart is thrown (at random) at a dartboard, what is the probability that a score of 50 or less will be recorded?

$2 \times 11=22$
$2 \times 12=24$
$2 \times 13=26$
$2 \times 14=28$
$2 \times 15=30$
$2 \times 16=32$
$2 \times 17=34$
$2 \times 18=36$
$2 \times 19=38$
$2 \times 20=40$
$3 \times 11=33$
$3 \times 12=36$
$3 \times 13=39$
$3 \times 14=42$
$3 \times 15=45$
$3 \times 16=48$
$3 \times 17=51$
$3 \times 18=54$
$3 \times 19=57$
$3 \times 20=60$


