Mathematics in the 21st Century

A very speculative look at how school mathematics might develop in the coming century.

Introduction

I am interested in some of the technological developments which are now available and wonder just how they will affect the educational scene and, in particular, the teaching of mathematics.

The main technology of course is that which involves computers, the internet, the World Wide Web, cyberspace and other buzz-words. What will it do for us? What can it do for us? There are probably as many ideas about its possibilities and how they can be developed as there are people prepared to think about it, and so this is just another speculation on why and how it might turn out.

First I should state my firm belief that computers (and all that is attached thereto) are going to play an increasingly larger role in the education system but I do not believe that they will ever replace the teacher, no matter how 'clever' they become. The teacher/learner interface requires the involvement of persons. Social interaction is as important to the educational process as any 'mechanical' learning that takes place.

My interest in this arose from some work on 'specialised calculators' which I first developed for my own convenience and use. Later I made them available publicly and was pleased, but rather surprised, at how much they were taken up and used by the world at large. Clearly there was an audience for this sort of thing. It was only then that I began thinking about such calculators in educational terms. This led, in turn, to speculation on how they might affect mathematics, in relation to both content and delivery, in schools. This paper is the result of that speculation.

Frank Tapson
July 2001

At the beginning of the 20th century there was some argument going on as to whether logarithms should be taught and used in schools, to help simplify the work of calculation. The principal arguments in favour were that it was quicker, and also that those who had difficulties with long multiplication and division would have an 'easier' method available to them. The principal arguments against were that accuracy would be reduced and pupils' abilities to deal with the 'necessary' algorithms of arithmetic would be lost. Sound familiar?

By the end of the 20th century, logarithms had come and gone (together with slide-rules) and the argument was then about electronic hand-held calculators. And we start the 21st century with that argument still going on.While there is no doubt in my mind about the eventual outcome, the century could be at least one-quarter gone before the argument even looks like dying out. And now we have to consider the computer, surely that must have some impact on the teaching of mathematics? It ought to. Indeed it must. In fact I believe it will have an impact on the entire mathematics curriculum, leading to a re-think on just what it is we try to do in the name of mathematics. But first an analogy which, in the general way of such things, gives the gist of my argument.

“Give a man a fish …

… and you feed him for a day. Teach him how to fish and you feed him for life.” This oft-quoted Chinese proverb just about sums up the basic idea of education, and we cannot really argue against the philosophy it expresses. However, especially in mathematics, it is in the application that we seem to go wrong. We insist on teaching him how to make the rod, line and hook, find the bait, and then catch the fish. (Unfortunately we do not teach him how to cook it.) Which is fine for running an SAS-style survival course, but we are working within a highly-civilised technologically-dependent culture, and the analogical equivalent of “making the rod etc” is inappropriate. The man could now, very reasonably, expect to be given the rod and shown how to use it!

How did we get here?

I often wonder if many teachers of mathematics realise just how lucky they are in their choice of subject. Very rarely are they called upon to justify its presence in the curriculum to their audience of both pupils and parents. How did mathematics get to this pre-eminent position, second only to English (well, reading and writing) as a subject that is somehow necessary to living? I speak only of mass-education here, which started in the middle of the 19th century. A different set of arguments guided the development of mathematics for the privately educated during the previous centuries.

When compulsory education started, arithmetic could clearly be seen to be essential, and especially so to those who were experiencing the Industrial Revolution. It is worth noting that the same period saw the growth of the Mechanics’ Institutes which provided the much needed technical education for the vast number of apprentices and improvers created by that same revolution.Geometry came into the mathematics curriculum towards the end of that century, and its (almost) extinction was mourned by the end of the next century. Trigonometry came in about the beginning of the 20th century and algebra a little bit later. Statistics did not become a topic of any note until after the middle of the 20th century. Calculus also bobbed in and out during the latter half of the 20th century. And then there was “new maths”.

All of these are very broad generalisations of course. There were wide disparities in the different sorts of schools, and no National Curriculum, but the above, very brief, historical synopsis is intended only as a rough guide to what has happened. A reminder also that things have not “always been like that” since time immemorial, and “golden days” only ever exist in retrospect!

Mathematics in our Culture

There is an ambivalence in the attitude of most adults to mathematics. On the one hand we are all too used to hearing “I was never any good at maths when I was at school”, while on another occasion we hear how important it is that their son or daughter gets a good grade in maths. This is not to say that these two expressions of attitude are contradictory on the face of it, but certainly they are if the ease with the first statement is uttered is taken into account. (How many people would volunteer the equivalent remark about their ability, or lack thereof, in relation to reading and writing?) So everyone ‘knows’ that the general ability of the world at large to do maths is poor, and we are reminded of it almost daily by the various media. Why then, is there this perpetual hope that the next generation will make some quantum leap in their understanding of mathematics?

And why is it important that their offspring should get any grade in maths, never mind a ‘good one’. A major contributory factor all too often is to do with it being (seen as, or actually) needed as an entry qualification to the next stage of the pupil’s life. Many teachers are familiar with the scenario of a pupil who is good at subject X (which is non-mathematical) who wishes to pursue it at a higher level but who requires a maths qualification in order to be admitted. And this qualification is only required because places at the next level are limited and an easy way of sorting out the candidates is to ask for a maths qualification regardless of whether it is appropriate or not. And then there are employers who insist on setting their own (usually very poor quality) test in mathematics. This is what one writer has described as the “gatekeeper function” of mathematics. Using it as an intelligence test. What an awful reason for doing all that work! No wonder there is generally a universal dislike of the subject.

One of the perennial arguments advanced for mathematics involves references to things like ‘education’, ‘transferable skills’, ‘developing logical arguments’ and ‘help in understanding other subjects’. These seem to be much more a matter of faith than of research and reasoned argument. Were not similar things once said on behalf of the classics? If anyone had said at the beginning of the 20th century that the classics would be just about non-existent as an educational subject by the end of the century, how many would have believed it? But it has happened. It is worth noting that every time some attempt is made to revive the classics there is a re-packaging in an effort to make it more palatable. However, it is no good, people do not see it as being appropriate.

Do you need an engineering background to drive a motor-car? Is a knowledge of electronics a pre-requisite for programming a video-recorder? Does even switching on a light need you to know the basics of electricity? The list is endless. It very apparent that living in the modern world would be impssible for all but a few if all those questions had to be answered ‘yes’. Following on from that, about the only school subject which can qualify for the word ‘necessary’ is that concerned with reading and writing, all the others are extras. Desirable or useful extras maybe, but mathematics is one those other subjects and should take its place in the discussion about what should be in the menu when the overall curriculum is written.

What then might be done to make some mathematics more accessible and more appropriate to our modern lives?

Maths for *tOP*

tOP is the acronym used here to represent *"the **O**rdinary **P**erson"*.

This embraces the vast majority of the population (at least 90%) who have no need of a 'full' mathematical background, such as we insist upon now, in order to enjoy a 'full' life.

So, how much maths does ***tOP*** need in order to make some sort of sense of the world around. Not much, and certainly nothing like the vast quantity of it that currently forms the basis of a typical school mathematics curriculum. ***tOP*** only needs to have

- a degree of numeracy
- some spatial awareness
- an understanding of data presentations.

Which in traditional terms is

- Arithmetic
- Geometry
- Data handling

The simplest possible summary of the content needed and purpose of these is

Arithmetic The main thrust of this should be concerned with how numbers 'work' together, and their relationships, especially in the matter of their relative sizes. Algorithms for doing 'heavy' arithmetic would be consigned to the history books** and calculations done by calculators.

Geometry This would be qualitative and knowledge-based, and much less quantitative than it is at present. So, identifying the basic shapes and being aware of their properties and relationships would be the prime purpose. While measures and measurements must remain, all calculations for purposes of mensuration** would be done using specialist calculators.

Data handling The overwhelming purpose of the topic is to help ***tOP*** gain some understanding of the enormous amount of data which is now available and handed out daily. And understand not only what it is saying, but also what it is not saying, its limitations as a tool for prediction, and how to assess its credibility.

In short,

Do less but do it better and make it appropriate!

** This does not mean such things would never be looked at. Just as now, there would always be room for topics of interest to be looked at and discussed at appropriate times and with the right audience, but these would not be a part of the basic curriculum.

And now, having made so much mention of calculators, it becomes necessary to spell out some detail about those.

Yes calculators, but which calculators?

Say ‘calculators’ to most people and they will immediately think of the hand-held variety and its use for doing basic arithmetic. (Not much more than + – × ÷ √) As remarked earlier, even their place in the teaching of mathematics has not yet been fully accepted and arguments rumble on. Then there are the scientific, graphics and programmable varieties, at least one of which no one doing ‘serious’ mathematics would dream of being without. But all of those, whatever their level, are only devices for doing computation. The operator has to provide the mathematics and, after manipulating it into a suitable form, uses the calculator only to deal with the tedium of the calculation. Even if the move is made up to something more powerful running on a computer (Derive, Mathematica etc) the steps are the same, do the mathematics and then let the program solve the equation or whatever.

tOP will not need anything more than a basic calculator. Indeed, within a short period of taking the last examination, he or she will probably have forgotten how to use the other function keys on a more sophisticated model. What ***tOP*** will always be able to use though is a calculator which allows the problem data to be input almost as it stands and return the answer. This implies not one general purpose calculator, but a whole spectrum of calculators, each dedicated to solving a particular type of problem.

Let us look at two examples of a dedicated calculator in use. The first example is based on a possible situation for ***tOP*** and the other is related to what, initially, must be a curriculum scenario.

Example 1

People now are much more knowledgeable about, and interested in, matters concerning their health and general standards of fitness, and one measure is that of Body Mass Index (BMI). This is given from

$$\text{BMI} = (\text{weight in kilograms}) \div (\text{height in metres})^2$$

and most would have little trouble in working that out.

The subject can look at the resulting value in the appropriate table or chart for the age and sex, to see how they compare in terms of size with ‘norms’. (At this point errors in the calculations stand revealed as being ‘off the chart’.)

That part is easy. Next they might wish to think about changing their BMI, getting it closer to some ‘ideal’ value. Clearly they can do nothing about their height, but they can alter their weight. So, what weight should they aim for?

Oh dear, the simple transposition called for in that calculation is beyond many pupils even when they are still at school, and a few years later only a few can handle it at all.

Now it requires very little imagination to visualize how a calculator to deal with that situation might work. Three entry boxes (one each for BMI, weight, height). Key in **any two** values and the third will then be calculated and displayed. As a bonus, the calculator could allow entries to be made in pounds and feet and deal with the necessary conversions as well. Moving up a step, input age and sex and the matching table or chart could be displayed. Simple! Trivial? Maybe, if you are comfortable and confident with some elementary mathematics, but not many people come into that category.

Example 2

Appendix 1 provides a worksheet with a set of conventional-looking questions concerning the pyramid. It has been divided into 6 sections, and the following development can be traced through them.

Section A deals with what a pyramid is (the geometry)

Section B concerns only the volume.

Section C requires the use of Pythagoras

Section D invokes trigonometry for finding angles

Section E continues with trigonometry but now finding lengths

Section F needs some ‘serious’ algebra (or ‘trial and error’)

So each section needs work to be done in a major topic-area of mathematics and the gradation of the sheet overall can clearly be seen as a progression from a low level to a high level. In fact the last two questions are very high level indeed (for school mathematics) since, if they are to be tackled algebraically, a cubic equation has to be formed and solved. The questions involve a variety of mathematical knowledge and techniques, all based on the well-known shape of a pyramid.

Now consider having access to a calculator that deals specifically with the pyramid. All of those questions can be answered by almost anyone who can read.

Where are they?

So where are these calculators to be found?

On the Internet or, more precisely, the World Wide Web. These dedicated calculators are quite small, typically 10 to 30 kilobytes, so they can be downloaded in seconds.

There already several thousand available and the number is increasing all the time. The quality is extremely variable and there is much duplication, but a brief look at some of them should convince any viewer that the provison of dedicated calculators is already well underway.

A good starting point is from this site -

Start at the Home Page:

www.ex.ac.uk/trol/

select => Specialist Calculators on Line = *SCoL*

That gives immediate access to over 50 dedicated calculators** (indexed by application) and, perhaps more importantly, at the bottom ot the page there is a link to a central location (in the USA) which attempts to list and link all on-line calculators world-wide. It is a very ambitious project! Currently it lists over 15,000 of them.

** These are the ones, mentioned in the Introduction, which led to the writing of this paper.

A Calculator Initiative

As mentioned, there are several on-line dedicated calculators available from this site. But in addition, a set of worksheets has been written especially for use with these calculators. It can be found also from the TROL home page mentioned earlier

select => Teacher Resources on Line = *TRoL*

select => Calculator Exercises

In addition to the many worksheets there are notes (for teachers) outlining the development intended so that pupils gain an awareness of what is available, together with an ability to select and use the correct calculator and apply it to particular problems. And the worksheets should not be used without reference to those notes.

A study of the notes, together with a look at a few of the calculators and a rough appraisal of the content of the worksheets, should make it apparent just how simple, yet so powerful, such calculators are. And these (first version) calculators are not particularly sophisticated, it is not difficult to think of other features which could be added, though care needs to be taken that their simplicity of use is not lost.

Just reflect on the background mathematics that is driving these calculators,

- formulas, used directly or transformed
- simultaneous equations made and solved
- linear, quadratic and cubic equations solved
- iterative procedures carried out (up to 1000 loops)
- answers given to requested number of significant figures

and all unbeknown to the user (why should it be?). All the user has to do is interpret the problem and put the figures in the correct boxes and then extract the answer needed to the question asked. Returning to the fishing proverb and analogy described earlier, the man has now been given the rod and shown how to use it!

The Serendipity Factor

Another benefit of these calculators to be noted is the amount of additional material they carry. There is not just a series of boxes for entries and answers, but also quite a lot of information about the subject of that calculator. This provides not only needed or sought for background, but also plenty of opportunity for serendipitous learning.

> **serendipity** *n. (18th cent.)* the making of happy and unexpected discoveries by accident, or when looking for something else.
> Coined by Horace Walpole in 1754. Formed on the fairy-tale *The Three Princes of Serendip*, the heroes of which "were always making discoveries by accident, of things they were not in quest of".
>
> [O.E.D.]

What about the rest?

The argument so far has concerned only ***tOP***. What about the other 10%?

This group would be those for whom it could be claimed that they really did need to have a mathematical background because the were going on to be mathematicians, scientists, engineers etc. Unfortunately the arguments will be confused at this point by all sorts of claims made for those that 'need' mathematics as a background for their work.

A fresh and objective look needs to be taken at the situation, and hard questions asked and answered. Do they 'need' the whole of the mathematics curriculum or only part of it? Is it 'necessary' or only 'useful'? Can it, or will it, be supplied some other way, perhaps by training or by being readily available at the press of a key?

> My own belief is that the figure of 10% will turn out to be much lower (less than 5) eventually.

So, given that we do not need to crucify the entire school population in the name of mathematics just so that 10% of them can be properly prepared for their career, what can we do about it?

The underlying problem is that of identifying the 10%. One way of dealing with this would be to have a 'tapered' curriculum (in both content and time) for mathematics, which (at secondary level) engaged ALL pupils in the first year, say 80% in the second year, 50% in the third year, and making mathematics optional after that, as it already is for most subjects.The curriculum for ***tOP*** would initally run within, and later beside the main curriculum for ALL pupils in ALL years, and be much much smaller in both content and time (no more than one period a week).

Other pressures for, and benefits from, change

So far we have looked only at what could happen to the mathematics curriculum because of what can loosely be classed as "mathematical factors". But there are other, broader factors at work (there always are!) which we should bear in mind. As they are only peripheral to the main thrust of this paper they are only outlined briefly. They come under two headings.

1. The staffing problem

The full mathematics curriculum clearly requires adequately qualified and trained teachers to deliver it. It is also very clear that we do not have enough of them. I am not referring here to 'local' difficulties of recruitment which happen for all sorts of reasons, but to an overriding factor that ensures we cannot ever have enough for simple background reasons.

Any society has a need for people to be doing all sorts of jobs. To avoid needless controversy, or giving offence, we will not name any jobs here. It is sufficient to say that we generally accept that some jobs are 'more highly regarded' than others, and that implies an ordered list of jobs.

We can do a similar exercise with qualifications, and also try to visualize how the number of people available, having each of those qualifications, would be distributed. Presumably we would expect to to find a small number at the very top and, hopefully, a small number at the very bottom, with the rest distributed somehow in between.

Now market forces come into play. The people controlling the jobs not unnaturally, but sometimes inappropriately, wish to select from as high up the qualifications list as they can get away with. (Set too high a target and you get no applicants.) But the jobs have to be done by someone and so, eventually, in order to get them you just have to lower the level of acceptable qualification.

Unfortunately we have raised the required level of ability needed to teach the full mathematics curriculum adequately without being able, at the same time, to raise the level of qualification needed for entry to the job. In fact there are many who consider the latter has gone down!

A reduced mathematics curriculum, with its consequent need for less specialists to deliver it, would enable the balance to be restored. Or at least provide a much better chance of doing so.

2. The 'more is better' syndrome

It seems to have been a basic tenet of compulsory education since it started that 'more is better' as exemplified by the raising of the school-leaving age at intervals. (The drive for increased admissions to higher education is another aspect of it.) But have we gone too far? Have we passed the optimum?

Certainly there are several pupils who spend their last year in school full of resentment at having to he there and seeing little point in it. (Their teachers have similar feelings, but they at least are paid for suffering it.) They clearly are not going on to higher education so, is there any need for that last year? Whatever they are going to extract from the educational system they will have got.

Let them go a year early. With conditions. Very reasonably, we would like all pupils leaving school to have a basic standard of literacy and numeracy. So, let those who wish to do so take a basic test which, provided they pass, entitles them to leave one year early. What an incentive to learn that would be! And another, slight, easing of the staffing problem.

How might it happen?

It is clear that we are not talking about overnight changes here. It will take many decades. These are changes which cannot be brought about by decree. A change in the perceived 'need' for mathematics in the culture is necessary and that can only happen gradually. It is a generational change which will come about as more and more people come to see the inappropriate nature of what is delivered at school when set against what they actually 'need' in life. And even more so, when they realise that, should the need arise, the tools are available. The problems that occupied hundreds of hours at school, learning to solve them from first principles (and so often failing) can in fact be solved effortlessly. "What was the point of it all?" they will ask, with increasing vigour.

As with all manner of innovations in the past, some impetus will be provided by the more 'adventurous' teachers First by individuals, then by groups of like-minded persons, and then it develops into a small-scale project, and then …

In short, evolution and not revolution.

And finally …

the biggest influence for educational change in this century is most likely to be the internet and particularly that aspect of it known as the Web. It means having access to the biggest library of information currently available. No, it does not contain everything, but it is heading that way. It is one of the great strengths of the Web that it allows one individual, with a specialisation or enthusiasm in a particular field, to put together a source of information that is immediately available world-wide. But then the sheer size of the whole thing poses its own problem - finding what you want.

However, in my capacity as a Web-manager of a particular site for several years, another problem which concerns me is stability. I am often informed of a 'good' site which it might be worth mentioning and putting in an appropriate link. I have done so several times, only to find a little later it is no longer there. A few experiences like that eventually make one very reluctant to link up with (mainly) individuals who abandon their sites for all sorts of reasons - moving it elsewhere, failing to pay their rent, losing interest, dying! Even organisations can 'disappear' as well. This is a pity, as well as an annoyance, because some really good sites can be found and noted as heing useful only to find, when you need it, that it no longer exists. There is a lack of stability.

I have a hope that universities might fill a role in helping to provide some much needed assistance with these problems. After all they were once the principal repositories of knowledge (the Great Library at Alexandria was possibly the first) and perhaps they could regain that position. A recent announcement by the Massachusetts Institute of Technology that it was going to make all of its course materials freely available via the Web as 'a blow against the privatisation of knowledge' was a hopeful start to the century. Might it help "to encourage the others"?**

Could this be the century of the 'open knowledge' revolution? Teaching our pupils to handle that could be our biggest task.

** Mind, in the UK I am not too hopeful. The way the universties are funded, their 'political' control, the Research Assessment Exercise to which they are subjected, are all calculated to restrict their freedon and their ability to be able to deal with the sort of work for which universites are ideally suited.

Some appropriate quotes

It was the mathematician, Leibnitz (1646-1716), who invented calculus, who said,

> "**It is unworthy of excellent men to lose hours like slaves in the labour of calculation.**"

Professor Henry Baker, when he was President of the Institute of Mathematics and its Applications (IMA) a few years ago, was quoted, in a newspaper interview, as saying

> "**Maths is still taught in the abstract rather than with reference to applications …**
>
> **… it is perpetuated by a minority of mathematicians who are a bit insular with life skills that aren't all they might be.**"
>
> and, later in the same report,
>
> "**some verge on the Luddite when it comes to technology**" he said with a smile.

Is it possible that Izaak Walton knew of the Chinese proverb "Give a man a fish …" when he wrote in his book *The Compeat Angler*

> "**Angling may be said to be so like mathematics, that it can never be fully learnt.**"

The Cockcroft Report (1982) was considered to be a most excellent report (at the time) and was often cited as an expositon of what mathematics in schools should concern itself with. Most quoted of all was its paragraph 243 which highlighted the framework of a 'good' lesson.

However, I have always thought that its best bit was the succinctly expressed piece of philosophy of what our teaching should really be about, in paragraph 34

> "**Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed, whether this be little or much**"

Pyramids

A graduated set of problems

In all of the following questions, the word "pyramid" means a right, square-based pyramid.

.Section A

1. What is the meaning of "right" in the description above?

2. Describe some other sorts of pyramids.

.Section B

3. A pyramid has a base edge length of 3.4 cm and a perpendicular height of 11.4 cm. What is its volume?

4. A pyramid has a volume of 158.4 cm^3 and a perpendicular height of 13.2 cm. What is the length of its base edge?

Section C

5. Find the length of the slant edge of a pyramid which has a base edge of length 9.36 cm and a slant height of 5.95 cm.

6. What is the perpendicular height of a pyramid having a base edge 13 cm long and a slant height of 9.7 cm?

7. A pyramid has a perpendicular height of 13.5 cm and a slant edge length of 18.4 cm. Find its volume.

Section D

8. Find the angle between one triangular face and the base, for a pyramid having a base edge of length 3.6 cm and a perpendicular height of 7.4 cm.

9. The angle between a triangular face and the base of a pyramid is 63° and the slant height is 17.1 cm. What is the perpendicular height?

Section E

10. A pyramid has a volume of 108 cm^3 and an angle of 74° between its base and any of its triangular faces. Find its perpendicular height.

11. What is the length of the base edge of a pyramid which has a slant edge of length 14.2 cm and an angle of 52° between its base and any triangular face?

Section F

12. The slant edge of a pyramid measures 25.2 cm and it has a volume of 2438 cm^3. What is the length of the base edge?

13. A pyramid has a volume of 240 cm^3 and a slant height of 13.3 cm. Calculate its perpendicular height.

From the Papers

Letters to the The Times
14th July 1998

Sir, To have seen our Secretary of State of the DfEE struggling, on television yesterday, with nine times eight, is an amusing sidelight on his edict that calculators will be banned for under-eights and that "setting tables to reggae music … could be made fun".

In 1928 I was in a primary class of some 40 five-year-olds and we learnt our tables to a tune. And as a result, unlike Mr Blunkett, I can give an instant answer.

But all this is quite irrelevant to the world in which we now live and work. I can think of not a single instance in the last 12 months, among the many hundreds of people with whom I must have had contact, when mental arithmetic of that kind was either used or indeed needed.

Will Mr Blunkett please tell us precisely why he now deems it necessary for our youngsters to do that kind of mental arithmetic.

Yours faithfully,
BRYAN THWAITES
July 9.

The above brought a heavy response, giving examples where immediate recall of number facts was of benefit in ordinary life.

Sir, I would like to remind Mr. Blunkett that a well trained parrot can say the times tables.

I may only be a student teacher and, as such, perhaps I have not seen the benefit of chanting times tables. It seems to me that children may well know that four times five is twenty, but change the sum around to five times four and they will not answer.

The chanting of times tables has been condemned since the start of the century. Have we not moved on from the heavily didactic methods of teaching that Mr. Blunkett is now proposing? Can he please allow teachers to teach children how to understand - not just to chant.

Yours sisncerely,
GARETH KIRKPATRICK,
July 9.

24th July 1998

Sir, I applaud the intention of the Government's numeracy consultants to emphasise mental arithmetic in primary schools (report July 9) but, unlike David Franklin (letter, July 17; see also letters, July 14), I would give calculators to kindergarten pupils.

There is no contradiction. Indeed, there is a symbiosis. Excellent mental arithmetic skills are needed, not just to perform calculations for which calculator use would be silly, but also to make it possible to determine whether results from a calculator are reasonable. Of course, mental arithmetic proficiency must be tested without calculators.

The ability to perform mental arithmetic for one and two-digit calculations will achieve the same, or better, numeracy and mind training than was supposed to have been attained with pencil and paper. Mental arithmetic, with judicious use of calculators, will obviate the need to teach obsolete pencil and paper skills at all and will allow much more mathematics to be taught in primary school than is now possible.

Sincerely,
ANTHONY RALSTON
July 17

29th December 2001

Sir, I disagree with the view that we don't have enough maths teachers (report, December 26). Too many maths teacher are misemployed trying to teach children with no mathematical ability, no employment requirement to learn mathematics and no interest in learning it.

I write as one who, after 50 years in industry and university, has been brought out of retirement in the vanguard of a Dad's Army of maths teachers. The problem is classifying everything from learning multiplication tables to the solution of differential equations as a single subject. If we recognised that basic arithmetic is really all that most people need to get by in a world of pocket calculators, the problem would be solved in a trice.

Primary school teachers are doing admirably with the new numeracy project; very little extra maths is needed from the age of 11. Geometry, algebra and trigonometry can be introduced in the first years of high school, by which time the children who will be needing to continue with maths for career purposes, and indeed able to handle maths, will have become clearly visible.

There is a perception that maths is harder than other subjects; it's true - for most children. My heretical solution is to stop wasting pupils' and teachers' time, release them from the agony of the impossible and significantly reduce the perceived need for the teaching of maths. We could probably cut the number of pupils taking maths after about 13 by some 80 per cent, with a consequent reduction in the requirement for teachers, at the same time producing a big jump in quality.

There is no shortage of maths teachers, rather an abundance of children forced to learn mathematics.

Yours faithfully,
NORMAN SANDERS,
December 27

From the Papers (*continued*)

Letters to the The Times 31st July 2002

Sir, Complaints from employers and universities have raised the possibility of reorganising (again) the mathematics syllabus in schools, and dropping algebra from the GCSE level (report and leading article July 24).

I used to be a teacher of maths and physics and sympathise entirely with people who find the subject difficult. The range of ability in maths is, I think, more pronounced than in any other subject. If a youngster has ability, it can be developed very rapidly to a high level and it can be very frustrating to such a person to be held back by the needs of other pupils - I speak from experience.

Everybody needs some mathematical skills, but they will be different depending on what the youngster goes on to do on leaving school. A hair-dresser will be unlikely to require a knowledge of algebra but an engineer certainly will, and a great deal more besides.

This is a situation where one size does definitely not fit all. The country badly needs its mathematicians.

As a former deputy head teacher I see no reason why it would not be possible to stream a year group with a high-flyers' class, several classes for those of average ability and special help for the ones who have the greatest problems. I used to teach the last type of class, and yes, algebra was on the syllabus, which was a great trial to us all. But once the GCSE exam was out of the way, we enjoyed maths again because I taught them about mortgages, having to pay interest on loans, how to measure up a window to make curtains, income tax and other practical problems.

Providing suitable and enjoyable mathematics courses for all pupils will reduce truany by those who hate the subject, while making sure that the future generation of mathematicians is not held back in the formative years.

Yours faithfully,
OLIVE D. HOGG
July 28.

3rd August 2002

Sir, Mrs Olive Hogg (letter, July 31) is absolutely right to say that at least basic mathematics must be taught to all young people.

However, the essential life skills she mentions - mortgages, loans, measuring curtains, income tax, etc - must also be part of the maths curriculum, as there is no such thing as "after GCSE exams" in the state system these days. The students leave at half-term, turn up for the exams and, in many cases are not seen again. Far more practical use could be made of the after-GCSE examination period, especially now that the students take continuous public exams in the AS and A2 years.

Yours faithfully,
FRANCES CHARLESWORTH
July 31.

Sir, The proverbial hairdresser in Mrs Hogg's letter may not "use" algebra, but nor, I doubt, will he or she "use" history, geography or science outside of conversation about the weather.

These subjects are not on the curriculum because they are necessarily vocational, but because they help children to practise skills which are widely applicable throughout every area of life.

We teach creative writing to all, not just budding writers, in order that all children learn to express themselves more fully. We teach algebra to all, not just budding mathematicians, in order that all children learn to think.

If that requires some changes to the style in which it is taught, so be it, but we would be denying children a precious skill were it to be totally removed from the GCSE.

Yours faithfully,
K. BREWIN
(Maths teacher)
July 31.

Sir, Now that it has been suggested that algebra should be dropped from the mathematics GCSE, might I suggest that verbs be omitted from the modern languages one. I have taught French and Spanish for almost 30 years and many of my students have found verb tenses a struggle.

Everyone knows that *un grand cafè crème pour moi et un croissant pour mon père* and similar phrases are sufficient these days for foreign travel. Anyone who can master such complex language deserves a grade B; anyone who could add *s'il vous plaît* would obtain a grade A, as this contains a verb.

Yours faithfully,
GEOFF BUCKLEY
July 31.

Appendix 3

Extracts from some official reports.

"In arithmetic, I regret to say worse results that ever before have obtained - this is partly attributable, no doubt, to my having framed my sums to require rather more intelligence than before: the failures are almost invariably traceable to radically imperfect teaching."

"The failures in arithmetic are mainly due to the scarcity of good teachers of it."

"Many who are in a position to criticise the capacity of young people who have passed through the schools have experienced some uneasiness about the condition of arithmetical knowledge and teaching at the present time. It has been said, for instance, that accuracy in the manipulation of figures does not reach the same standard which was reached twenty years ago. Some employers express surprise and concern at the inability of young persons to perform simple numerical operations involved in business."

"The standard of mathematical ability of entrants to trade courses is often very low. Experience shows that a large proportion of entrants have forgotten how to deal with simple vulgar and decimal fractions, have very hazy ideas on some easy arithmetical processes, and retain no trace of knowledge of algebra, graphs or geometry, if, in fact, they ever did possess any."

"There are indeed many adults in Britain who have the greatest difficulty with even such apparently simple matters as adding up money, checking their change in shops or working out the cost of their petrol. Yet these adults are not just the unintelligent or the uneducated. They come from many walks of life and some are very highly educated indeed, but they are hopeless at arithmetic…
During this investigation the firm impression has built up … that functional innumeracy is far more widespread than anyone has cared to believe."

"The extent to which … even an apparently simple and straightforward piece of mathematics could induce feelings of anxiety, helplessness, fear and even guilt in some of those interviewed was, perhaps, the most striking feature of the study."

> And there are many many more like that. All very worrying. But what must be cause for greater worry is to note when those particular reports were written. The first two in 1876, the next in 1925, the fourth in 1947, the fifth in 1979 and the final one in1982.
>
> *(All have have been taken from the Cockcroft Report of 1982.)*
>
> Even worse, things like that are still being written.

In short …
We have been failing to teach mathematics successfully to pupils in our schools for well over 100 years. Is it not time we acknowledged that failure and tried to something (different) about it?