A Formula Miscellany

Frank Tapson

Some thoughts on formulas and their usage, together with examples of more unusual applications and appearances.

What is the best format?

In the classroom we have become accustomed to certain formats for our formulas, and there might be good reasons for them being that way, but it does not mean that those formats are the most useful when it comes to applying the formulas in the work-place.

A good example is the usual formula for the volume of a sphere: $V = \frac{4}{3}\pi r^3$

Mathematically there are very good arguments for it, but what about someone who has a sphere (for real, not in a sum) whose volume has to be found, for some reason? Never mind that 4/3 at the front (which confuses many), how is the radius to be found? Not by direct measurement – that requires a special tool (called a spherometer). It is easy to measure the diameter however. So then divide that by 2 and the job is done. Well yes, but why not streamline the whole thing and reduce the number of steps needed (as well the chances for errors)?

What is wrong with learning and using the formula $V = \frac{\pi d^3}{6}$?

It is interesting to note that in those sports which require a spherical ball to be used, the rules governing the game which lay down the sizes of everything, specify not the radius of the ball, but its circumference. It is easy to see why. One can imagine the referee or umpire carrying a tape measure with which to check measurements if called into question, but carrying a pair of callipers as well would seem to be an unnecessary burden. *(If the volume were needed, what would the appropriate formula be?)*

Let's go into the woods.

"Woodman, woodman spare that tree, but tell me what its volume be."

A very common problem. Before trees are felled for making timber, they are usually measured and their volume calculated so that a good estimate can be made of their eventual value. After all, it may be that the final commercial value does not justify all the work.

Well that's easy: $V = \pi r^2 h$. Done! Wait, how does one find the radius of a standing tree? Finding the diameter is possible but would require a very large pair of callipers to be available. Using a tape to measure the circumference (called the 'girth') is much easier and then the formula becomes $V = \frac{C^2}{4\pi}h$

Now other practicalities take over. In the middle of a large group of trees, and with a need to use an estimated average for h, that π looks like a piece of fussy precision. $V = \frac{C^2}{12}h$ will do.

In fact, the 12 (or 13) can be changed into other (usually bigger) values to make an allowance for the type, age, condition, and bark thickness (= wastage) of the tree. And the woodsman's handbook contains tables of pre-determined values to allow that to be done.

Making it simple

or as simple as possible.

Negative numbers, and especially their use in doing arithmetic, are difficult enough for most pupils to deal with even while they are still at school, and some years later on when they are encountered in the course of work, the difficulties have not lessened even though there is now some real point in doing them. In fact, the job could depend upon it.

An example of this is to be found in the use of cosines. The cosine of an angle can vary in value from 1 to $^{-}1$ over the range 0 to 180 degrees. This creates a need to be able to work with negative numbers if the formula involves cosines.

This need can be overcome by the use of versines.

Versine (also written versin) is a contraction of ‘versed sine’.
It is defined as**:** versine $A = 1 - \text{cosine}A$.
First recorded usage was in 1827.

Over the range 0 to 180 degrees the versine of an angle varies in value from 0 to 2 so those angles can be handled without any ambiguity or the need for using negative numbers, and navigators are very glad of it. Navigators also use the ‘haversine’ (= half a versine) in their calculations.

The versine rarely (if ever) gets a mention in the classroom, but it would remove a few difficulties in the use of the cosine rule.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

becomes

$$a^2 = (b \sim c)^2 - 2bc \text{ versine } A$$

$(b \sim c)$ means the smaller is taken from the larger.

A small, but helpful, improvement

Some other applications of formulas

Wind Chill Factor

v is wind speed in miles per hour
t is air temperature in degrees celsius or °C
W_{cf} is wind chill factor in degrees celsius or °C

There is no universal agreement on a 'correct' formula but two are

$$W_{cf} = 33 + (0.45 + 0.29\sqrt{\boldsymbol{v}} - 0.02\boldsymbol{v})(\boldsymbol{t} - 33)$$

$$W_{cf} = 33 + (0.2673 + 0.1684\sqrt{\boldsymbol{v}} - 0.01135\boldsymbol{v})(\boldsymbol{t} - 91.4)$$

Reading Age of a Text

N is number of **words** in the text being analysed
S is number of **sentences** in the text
n_s is number of **syllables** in the text
n_1 is number of **1-syllable** words in a 150-word sample of the text
n_3 is number of **3-syllable** words in the text (excluding -ing and -ed endings)
R is reading age in years

There are several possible formulas. Some of the better known are

$$R = 25 - (n_1 \div 10)$$

$$R = [0.4\,(N/S + n_3)] + 5$$

$$R = (n_s/N \times 0.39) + 11.8N - 10.59$$

$$R = (0.0778N/S) + (4.55n_s/N) + 2.7971$$

Some interesting appearances of formulas

Examples of the applications of formulas to more complex situations, and what the newspapers made of them follow on the next pages. Some editing has been done on the text to shorten it a little while retaining the essentials of the description, but every care has been taken to copy the formulas, and all references to them, exactly as they were printed.

Not to be taken too seriously!

Other work concerning formulas

On the main *TRoL* menu under Exercises and Practice can be found Using Formulas which provides a comprehensive introduction to, and plenty of practice with, working with formulas.

On that same menu (near the bottom) Miscellaneous Papers can be found. Listed on that menu is a paper on the topic of BODMAS which gives some background on one aspect of formulas which has occupied pupils for many decades!

Formula for a win but is it cricket?

BRITISH scientists have solved a cricketing mystery that could help reverse the flagging fortunes of the England team. They have unravelled the secret of the world's most feared bowler's delivery.

With the help of computers, the laws of aerodynamics, wind tunnels and cricket balls borrowed from county clubs, they have worked out how the Pakistani bowlers achieved the deadly "reverse swing" that routed England's batsmen last summer.

The technique has been hailed by commentators as the most important innovation since players were first allowed to bowl over-arm more than 125 years ago. It defies conventional cricketing wisdom: as the bowler holds the ball as if to swing it one way, it confounds the batsman by curving in the opposite direction.

The secret lies in the movement of air around the ball, according to aeronautical engineers at the University of Hertfordshire who tackled the conundrum that has stumped the cricketing establishment.

"It's a case of applying aerodynamic laws that have been known for years," said Andrew Lewis, who headed the research. There is, however, a drawback for English cricketers: they need to bowl at more 85 mph to have the best chance of getting reverse spin.

Although the researchers found that "doctoring" the ball could improve performance, they say there are so many technical factors bowlers can exploit that illegal tactics are unnecessary.

These factors even include a formula in which the amount of swing equals $C_y \times ¼ \times p \times p^2 \times s$ divided by m, where C_y is the side force coefficient, *p* is the air density, p is the distance the ball travels, s is the ball's surface area and m is the ball's mass. For the less technically inclined, the researchers plan to simplify the findings and submit them to the MCC.

Many in the cricket fraternity are disturbed to see the intervention of science in the ancient game. "I know it works, but I am not interested in how and why," said Brian Johnston, the veteran BBC commentator.

"It's no fun trying to work out why a ball bobbles one way another. Cricket should be fun."

Motson scores on pitch

SCIENTISTS using a complex formula have calculated that the BBC's John Motson has the perfect voice for sports commentary, a pitch ahead of all his peers.

The sheepskin coat, flat cap, boundless enthusiasm and a true anorak's knowledge of the game's trivia garnered in 30 years behind the microphone have nothing to do with it. Motson is king thanks to the formula $SQ = P - OL/2 + (LV \times 2) + RA/2 + RH + (T\times l.5) - C/2$

Researchers at Queen Elizabeth Hospital in Woolwich, South London, studied pitch, overall loudness, loudness variability, rate, rhythm, tone and constriction to calculate an overall speech quality of 30, compared with a mere 22 for his nearest rival, Johnathan Pearce of Channel 5.

Mathematics has shown Motson's voice to have the pace of Michael Owen and versatility of Roy Keane. He employs twice the vocal range of an ordinary speaker and can talk twice as loudly or softly, all at double everyday conversational speed.

The equation where BBQ = perfect

MATHEMATICIANS at the University of Greenwich have devised a formula for barbecuing the perfect hamburger.

It's easy to avoid that charred black outside and bright red middle. all you need to know is the mass of the burger, its thermal conductivity, heat capacity, covective heat transfer coefficient, and the temperature of the grill.

What you really need to know, in plain English, is that double the thickness requires four times the cooking. Iceland, the food retailer, found in a customer survey that almost half of Britons burn food on the barbecue.

That, they reckon, represents £20 million of carbonised and largely inedible burgers, sausages, chops and chicken legs every year.

The store chain commissioned Dr Dilwyn Edwards, of Greenwich's School of Mathematics to provide a scientific answer. He came up with an indigestible portion of algebra, and his researches did not involve the grilling of a single burger. "It's a simple differential equation," Dr Edwards said yesterday in what sounded suspiciously like an oxymoron. "But the formula is only really useful if you have already founded the perfect cooking time for a burger of a specified size."

Dr Edwards noted that the mass, m, is proportional to Ax where A is the area of the burger and x is its thickness. If all other parameters remain the same - heat of grill, absence of sudden downpour, mood of cook and so on - then t, the total cooking time, is proportional to x2A.

In other words, if it takes 14 minutes to cook a burger 10cm (4in) in diameter and 1cm thick, it will 28 minutes to cook one 20cm across and 1cm thick, 56 minutes to cook one 10cm across and 2cm thick. Dr Edwards did not consider sausages. A whole new formula may be required for cylindrical items.

Till death or mathematicians do you part

From an American Association for the Advancement of Science conference

IF CHEMISTRY is the key to a happy marriage, the secret of divorce is maths: researchers have devised a mathematical model that can predict with astonishing accuracy which couples will part.

The formula, devised by a psychologist and two applied mathematicians, allows counsellors to forecast with a 94 per cent success rate which married couples will stay together and which are likely to part, providing a powerful new tool for marriage guidance.

The set of equations can give a prognosis of marital bliss or strife after the partners have been observed discussing a point of contention, such as sex, money, child-rearing or holidays for just a few minutes.

Marks are awarded for positive and negative interactions between the partners, and the ratio between these seems critical to the question of whether the marriage will last.

Jokes, flirtatious or affectionate gestures, smiles and an understanding tone of voice all win positive marks, while negative signals include eye-rolling, tutting and sarcasm. The "magic ratio" is five positive to one negative signal and any couple that scores below this is likely to be heading for trouble.

Dr Gottman said, "Before this model was developed, divorce prediction was not accurate, and we had no idea how to analyse what we call the masters and disasters of marriage – those long-term happily married and divorced couples. "When the masters of marriage are talking about something important, they may be arguing, but they are also laughing and teasing there are signs of affection because they have made emotional connections.

"But a lot of people don't know how to connect or how to build a sense of humour, and this means a lot fighting the couples engage in is a failure to make emotional connections. We wouldn't have know this without the mathematical model. It gives us a way to describe a relationship and the forces that are impelling people that we never had before.

"The maths is so visual and so graphical that it allows us to visualise what happens when two people talk to each other."

The study is the first to introduce rigorous mathematics into psychology in a manner that may clinically useful.

"When Newton invented calculus it put science on a mathematical foundation and physics really took off," Dr Gottman went on, "But psychology is a field that has lagged behind in using mathematics and there is no maths in social psychology.

"What we did is extract key elements into a model so it is interpretative and predictive. The maths is trivial, but the model is incredibly accurate."

The equation for the wife is:
w(t + 1) = a + rl*w(t) + ihw[h(t)]
Where w is the wife, h is the husband, t is time, a is the wife's state of mind when not with her husband, rl*(wt) represents the ease with which she changes her mind during a marital conversation, ihw is "influence function", h(t) is the husband's score after a 15-minute conversation, and w(t + 1) is how the wife reacts to her husband's remarks.

The equation for the husband is:
h(t + 1) = b + 2*h(t) + iwh[w(t)]
Where w is the wife, h is the husband, t is time, b is the husband's state of mind when not with his wife, rl*(wt) represents the ease with which he changes his mind during a marital conversation, ihw is "influence function", w(t) is the wife's score after a 15-minute conversation, and h(t + 1) is how the husband reacts to his wife's remarks.

(Editorial note: The two formula panels have been copied exactly as they were printed.)

Police put crime into the equation

POLICE in London have a new formula for cutting crime: they have paid mathematicians to find an equation that calculates where and when officers are needed.

The answer to why a policeman is never there when you need one is to be found in the sum NDb = (60% × Nc/Nt + 40% × Dc/Dt) × 17588 - the complicated equation that is now used by the Metropolitan Police Authority to allocate manpower and resources.

NDb signifies the number of officers to be allocated to each borough. The formula allocates 60 per cent of officers according to a borough's need - which includes factors such as day-time and night-time population, unemployment and economic deprivation. Nc is the need of a specific borough, and Nt is the need score across the whole of London.

The remaining 40 per cent are allocated according to the level of crime or demand: Dc is the demand in a specific borough, and Dt is the total demand; 17,585 is the total number of officers available.

$$C = \frac{C_o\sqrt{N}nad}{Mt(1 + a)}(2-d)$$

Scientists reach solution to chopstick challenge

SCIENTISTS at the University of Surrey have produced a formula to help dextrously challenged British fumblefingers to get to grips with chopsticks in time for the Chinese new year.

Three centuries after chopsticks were introduced to Britain, three out of five Britons admit to using a knife or spoon and fork when eating Chinese food.

The attitude has been diagnosed as a chronic case of consecotaleophobia, the dread of chopsticks.

The formula was devloped by Jim Al-Khalili and Qiang Zhao and quantifies the degree of comfort with which practioners can expect to tackle their tucker using the traditional oriental tools.

In the formula, C is the comfort level using chopsticks, with 100 as complete ease and one, denoting embarassing total incompetence. N is the number of meals previously eaten with chopsticks and T the time in seconds that it takes to get food from plate to mouth.

N, A, D and M stand for the shape, slipperiness, diameter and mass of the food, while Co is a constant incorporating unknown data such as the length of chopstick.

The secret is practice. Once one has eaten more than 1,000 meals with chopsticks, the scientist say, using them should be as easy as using one's fingers.

32 + 27 adds up to the perfect marriage age

AN EQUATION has been developed by top mathematicians that pin-points the correct age at which men and women should stop "playing the field" and get married.

According to the theory, 32 is when men should tie the knot, while the right age for women is 27. After these ages, the chances of finding the perfect mate begin to be out-weighed by the probability of spending years fruitlessly searching for the ideal partner and ending "on the shelf".

Dennis Lindley, emeritus professor of statistics at University College, London, said the theory supposes that a man has a set "window" for courtship. During the window – for instance between 16 and 60 for men – a man will typically start a relationship with a woman, and at some point must decide whether to marry her or break it off and gamble on finding a better mate. The danger, of course, is that nothing better comes along.

Some might say it is simply recognising the law of diminishing returns: as you get older, you have less choice. Under the formula, the age at which you should switch from searching for the dream wife to making a commitment to the first suitable woman is M.

This is calculated by taking Y (the age at which you started searching) and adding it to 1 divided by *e* (where *e* is 2.718) multiplied by X (the age at which you would expect to stop looking) minus Y. Each person can choose different values X and Y depending on when they start and expect to finish the hunt for a spouse.

Lindley said, "During the long run-in period when you don't marry, you are learning about the quality of the ladies.

"You may say, 'Well, this lady is wonderful but, wait a minute, she might not be the best'. So you go on, thinking that perhaps you are going to see a better one tomorrow. But you mustn't do this for too long, you must stop at some point"

$$M = Y + (1/e\ (X - Y))$$

M *= the age when it is right to consider marriage.*

Y *= earliest age for marriage: 16 for men and women.*

X *= possible latest age of marriage eg 60 for men, 46 for women*

e *= 2.71828, the base of natural logarithms. It is frequently used by mathematicians who know it as both an "irrational number" and a "transcendental number"*

From the front cover of

THE SUNDAY TIMES magazine

December 22, 1991

The Transport and Road Research Laboratory has devised a formula to enable you to calculate your mathematical chance of having a traffic accident next year. The formula you have to apply is this:

$$A_c = 0.00633\exp\{s+g\}$$
$$(1+1.6p_d)$$
$$(p_b+0.65p_r+0.88p_m)$$
$$M^{0.279}$$
$$\exp\{b_1/Ag+b_2/(X+2.6)\}$$